BASED ON NCERT TEXTBOOK & LATEST CBSE SYLLABUS FOR 2026-27

CLASS XI

Aligned with NEP 2020 & NCFSE 2023

# Mathmission

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01 COMPLETE THEORY WITH EXAMPLES
02 SUBJECTIVE TYPE QUESTIONS
03 COMPETENCY FOCUSED QUESTIONS
04 MULTIPLE CHOICE QUESTIONS
05 ASSERTION - REASON QUESTIONS
06 CASE STUDY & PASSAGE BASED QUESTIONS

O.P. GUPTA

INDIRA AWARD WINNER

BASED ON LATEST CBSE SYLLABUS
& NCERT TEXTBOOK

FULLY UPDATED FOR 2026-27
(WITH READING MATERIAL)

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# MATHMISSION

## FOR XI

MATHEMATICS
(Subject Code-041)


O.P. GUPTA
MATHS (H.), E & C ENGINEERING
INDIRA AWARD WINNER


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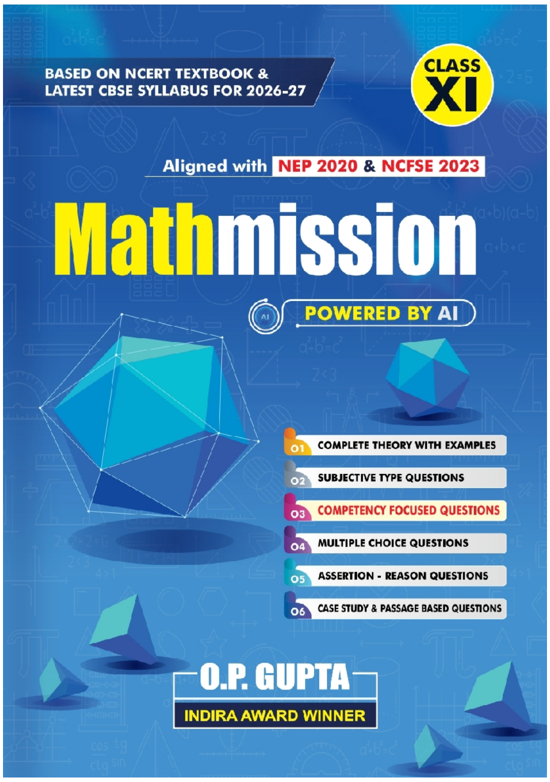


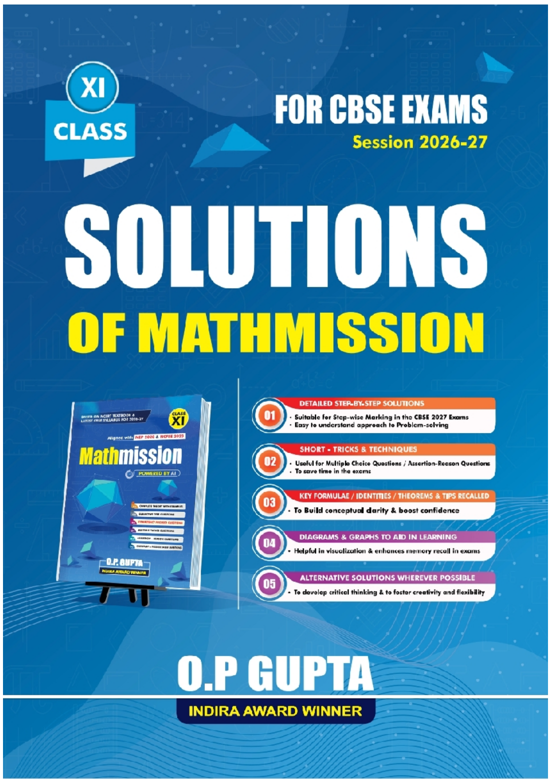


Following are the two Books for CBSE XI (2026-27) by O.P. Gupta, released in March 2026.

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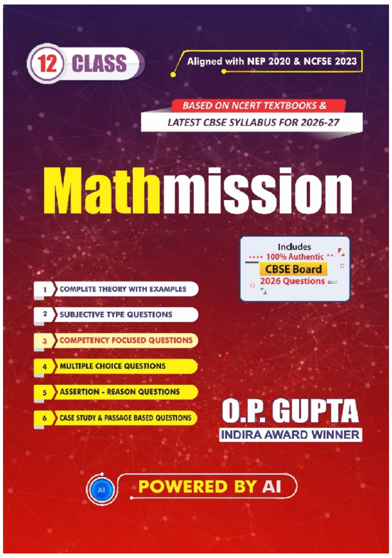


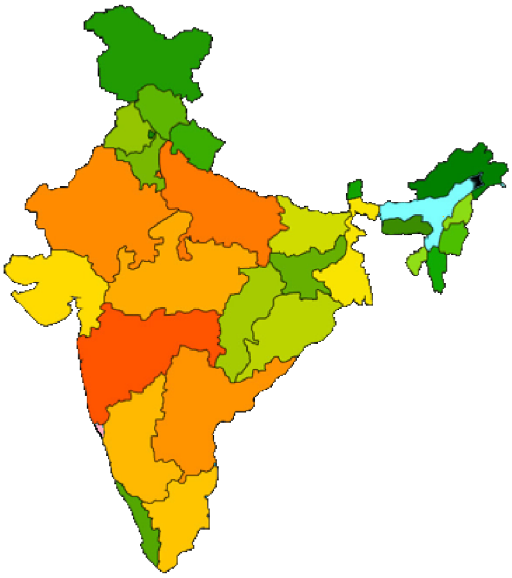

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## MATHMISSION @ FOREIGN LOCATIONS

- Oman
- Doha (Qatar)
- Saudi Arabia
- Dubai
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# TABLE OF CONTENTS



**✓ * Reading Material has been added in certain Chapters**

**✓ # Reading Material added by CBSE in Chapter 2 (Class XI) is to be mapped in Chapter 1 (Class XII). So, it is not given in this Book.**

**✓ ** Reading Material added by CBSE in Chapter 12 (Class XI) is to be mapped in Chapter 5 (Class XII). So, it is not given in this Book.**

***(as available on CBSEAcademic.nic.in)***

# CHAPTER 03

# TRIGONOMETRIC FUNCTIONS

*Mathematics, rightly viewed, possesses not only truth but supreme beauty.*

## OVERVIEW

**In this chapter, we shall learn**

- ✓ Definition of angle, positive & negative angles
- ✓ Different measures of angle
- ✓ Definition of degree, radian and relation in degree and radian
- ✓ Definition of trigonometric functions
- ✓ Signs of trigonometric functions
- ✓ Values of trigonometric functions at standard angles
- ✓ Domain & range of trigonometric functions
- ✓ Graphs of trigonometric functions
- ✓ Identities of trigonometric functions and their proofs
- ✓ Principal and general solutions of trigonometric equations*

## INTRODUCTION

Since the ancient days, trigonometry was first started in India. Its elements can be found in **Rigveda**. Many Indian mathematicians like Aryabhatta, Bhaskara I and II and Brahmagupta had got important results. All this knowledge first went from India to Middle East and from there to Europe.

Note that the word ‘trigonometry’ is derived from the Greek words ‘trigon’ and ‘metron’ and it means ‘measuring the sides of a triangle’. The subject was originally developed to solve geometric problems involving triangles. It was studied by sea captains for navigation, surveyor to map out the new lands, by engineers and others.

Currently, trigonometry is used in many areas such as the science of seismology, designing electric circuits, describing the state of an atom, predicting the heights of tides in the ocean, analyzing a musical tone and in many other areas.

In earlier classes, we have studied the trigonometric ratios of acute angles as the ratio of the sides of a right angled triangle. We have also studied the trigonometric identities and application of trigonometric ratios in solving the problems related to heights and distances.

In this Chapter, we will generalize the concept of trigonometric ratios to trigonometric functions and study their properties.

## IMPORTANT TERMS & DEFINITIONS

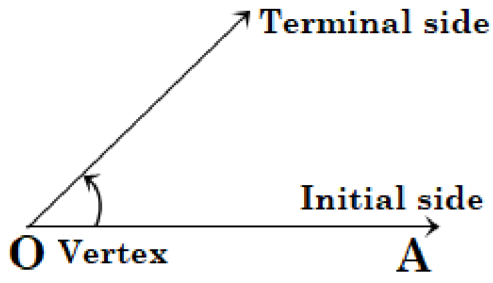
### 01. Angle in geometry

Angle is a measure of rotation of a given ray about its initial point.

That is, an angle is a figure formed by two rays having common vertex called as *origin.* The rays are called sides of the angle. The measure of the angle is the amount of rotation from the direction of one ray of the angle to the other.

The initial and final positions of the revolving ray are respectively called the ***initial side*** and ***terminal side*** and the revolving line is called the ***generating line*** or the ***radius vector***.

* **Principal and general solutions** of trigonometric equations are now in syllabus as **Reading Material** (from session 2025-26).




In the figure shown above, the ray OA is the initial side and ray OB is the terminal side. And they form **angle AOB** at the vertex O.

This angle is denoted by $\angle AOB$.

With each angle a number is associated and this number is called ***measure of the angle***. There are several units for measuring this angle and we shall study about them.

☑ In **geometry** an angle always lies between $0^{\circ}$ and $360^{\circ}$ and **negative angle** has no meaning.

## 02. Angle in trigonometry

The idea of angle is more general in trigonometry. It may be positive or negative and of any magnitude. We know that angles in geometry are confined only till $360^{\circ}$ which corresponds to one complete revolution by a wheel *say*. So it is quite obvious that the angle covered in two complete revolutions is of $720^{\circ}$ measure and in a quarter of revolution it is of $90^{\circ}$ and so on.

## 03. Units of measurement of angles

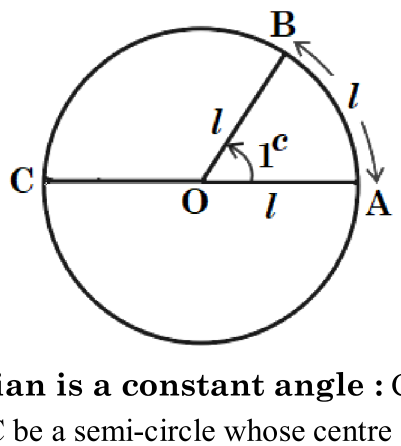
In geometry angles are measured in terms of right angle. In order to measure the smaller angles we need to introduce smaller units of angle. These are ***Sexagesimal or British System (Degree Measure), Centesimal or French System (Grade Measure)*** *and* ***Radian or Circular System***. Here we'll confine ourselves only to Degree measure and Radian measure. *Though you can expect a discussion about the* ***Grade measure*** *too in the class!*

**Degree Measure** : In the degree system of measurement a right angle is divided into 90 equal parts which are called as **degrees. Each part is equal to one degree.** Each degree is then divided in 60 equal parts called **minutes** and each minute is further divided into 60 equal parts called **seconds.**

A degree, a minute and a second are denoted by the symbols $1^{\circ}$, $1'$ and $1''$ respectively.

Thus, 1 Right angle $= 90^{\circ}$, $1^{\circ} = 60'$ and $1' = 60''$.

**Radian Measure :** The angle subtended at the centre of circle by an arc whose length is equal to its radius is called **a radian** and is denoted by $1^{C}$.




As shown in the adjacent figure, the centre of circle is O and its radius is of $l$ units.
So if the length of arc $AB = l$ units then, by the definition of radian given above, we have $\angle AOB = 1$ Radian.

☑ **Radian is a constant angle :** Consider the figure of circle shown above.

Let ABC be a semi-circle whose centre is at O and radius $l$. Let length of arc AB be equal to $l$.

Then by definition, $\angle AOB = 1^{c}$.

Now produce AO and let it meet the circle at C. Then AC is a diameter of circle and arc ABC is equal to half the circumference of the circle and $\angle AOC = 2$ Right angles $= 180^{\circ}$.

By our geometrical knowledge, we know that the angles subtended at the centre of a circle are proportional to the length of arc which subtends them.

*i.e.*, $$\frac{\angle AOB}{\angle AOC} = \frac{\text{arc AB}}{\text{arc ABC}} \quad \ldots(i)$$

$\Rightarrow \quad \dfrac{1^c}{180^\circ} = \dfrac{l}{\pi\, l}$ [As *arc* ABC is a semicircle

$\Rightarrow \quad 1^c = \dfrac{180^\circ}{\pi}$ ...(ii)

$\Rightarrow \quad 1^c = \dfrac{2 \text{ Right angles}}{\pi}$

$\therefore \quad 1^c = \text{Constant}$ .

**Understanding the $\pi$**

The $\pi$ is not a whole number, nor it can be expressed in the form of a fraction, and hence not in the form of a decimal fraction, terminating or recurring. The number $\pi$ has a value which can't be exactly expressed as the ratio of two whole numbers.
The value of $\pi$, correct to 8 places of decimals, is $\pi = 3.14159265...$

In fact, the fraction $\dfrac{22}{7} = 3.14285...$ gives the value of $\pi$ correct to the 2 places of decimals.

☑ Consequently we deduce that, $\pi^c = 180^\circ$ i.e., $\pi$ Radians $= 180^\circ$ .

☑ Also by (i) it can be easily deduced that $\theta = \dfrac{l}{r}$, if length of any arbitrary arc $AC = l$ which subtends an angle of $\theta$ radians at the centre O of the circle of radius r.

So, $\theta$ (*in radian measure*) $= \dfrac{l}{r}$.

## ☑ Relations in Different Measures of Angle

(i) *Angle in Radian Measure* $=$ (*Angle in Degree Measure*) $\times \dfrac{\pi}{180}$

(ii) *Angle in Degree Measure* $=$ (*Angle in Radian Measure*) $\times \dfrac{180}{\pi}$, where $\pi = \dfrac{22}{7}$.

☑ Following table can be consulted for a few frequently used standard angles:

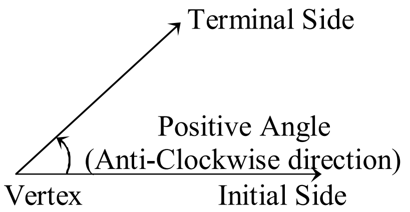
| **Angles in Degree** | $0^\circ$ | $30^\circ$ | $45^\circ$ | $60^\circ$ | $90^\circ$ | $180^\circ$ | $270^\circ$ | $360^\circ$ |
|---|---|---|---|---|---|---|---|---|
| **Angles in Radian** | $0^c$ | $\left(\dfrac{\pi}{6}\right)^c$ | $\left(\dfrac{\pi}{4}\right)^c$ | $\left(\dfrac{\pi}{3}\right)^c$ | $\left(\dfrac{\pi}{2}\right)^c$ | $(\pi)^c$ | $\left(\dfrac{3\pi}{2}\right)^c$ | $(2\pi)^c$ |

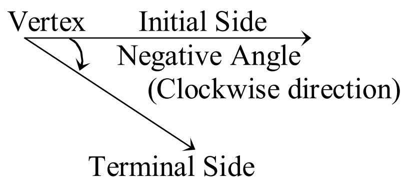
☑ In actual practice, we omit the exponent '**c**' and instead of writing $\pi^c$, we simply write $\pi$.
Likewise, instead of writing $\left(\dfrac{\pi}{6}\right)^c$, we simply write $\dfrac{\pi}{6}$. We do same for other angles.

- 1 Radian $= 57^\circ 17' 45''$ seconds
- 1 Radian $= 206265$ seconds
- $1^\circ = \dfrac{\pi}{180} = 0.01745$ radians (approximately).

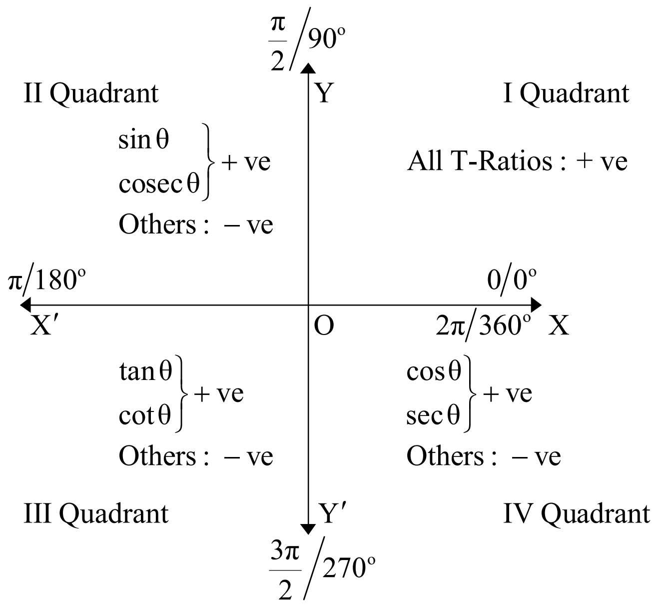
## 04. Sign of angles and Quadrants

An angle formed by **anticlockwise rotation** of the radius vector is taken as ***positive*** whereas the angle formed by **clockwise rotation** of the radius vector is taken as ***negative***.
For the clarification, have a look at the figures given below :




Consider $XOX'$ and $YOY'$ be two mutually perpendicular lines in a plane and OX be the initial half line. The whole plane is divided into four different regions namely $XOY$, $YOX'$, $X'OY'$ and $XOY'$. These regions are called quadrants and are respectively called $1^{st}$, $2^{nd}$, $3^{rd}$ and $4^{th}$ quadrants. The angle is said to be in any of these quadrants according as the terminal side lies in whichever quadrants. If the terminal side coincides with one of the axes then the angle is said to be a *quadrant angle*.
(Figure is given below).




If there is any angle $\theta$ which is not a quadrant angle and radius vector rotates in the anticlockwise direction in such a way that number of revolution doesn't exceed one, we have:

| | |
|---|---|
| (i) $0^\circ < \theta < 90^\circ$ | If $\theta$ lies in I quadrant |
| (ii) $90^\circ < \theta < 180^\circ$ | If $\theta$ lies in II quadrant |
| (iii) $180^\circ < \theta < 270^\circ$ | If $\theta$ lies in III quadrant |
| (iv) $270^\circ < \theta < 360^\circ$ | If $\theta$ lies in IV quadrant |

Also when terminal side coincides with $OY$ : $\theta = 90^\circ$
when terminal side coincides with $OX'$ : $\theta = 180^\circ$
when terminal side coincides with $OY'$ : $\theta = 270^\circ$
when terminal side coincides with $OX$ : $\theta = 360^\circ$.

**Type of angles**
- Acute angle : $0^\circ < \theta < 90^\circ$
- Obtuse angle : $90^\circ < \theta < 180^\circ$
- Reflex angle : $180^\circ < \theta < 360^\circ$

☑ Following table will be sufficient to give you an idea about the discussion we just have had:

| Angles $(\rightarrow)$ / T-Ratios $(\downarrow)$ | $\frac{\pi}{2}-\theta$ | $\frac{\pi}{2}+\theta$ | $\pi-\theta$ | $\pi+\theta$ | $\frac{3\pi}{2}-\theta$ | $\frac{3\pi}{2}+\theta$ | $2\pi-\theta$ **OR** $-\theta$ | $2\pi+\theta$ |
|---|---|---|---|---|---|---|---|---|
| sin | $\cos\theta$ | $\cos\theta$ | $\sin\theta$ | $-\sin\theta$ | $-\cos\theta$ | $-\cos\theta$ | $-\sin\theta$ | $\sin\theta$ |
| cos | $\sin\theta$ | $-\sin\theta$ | $-\cos\theta$ | $-\cos\theta$ | $-\sin\theta$ | $\sin\theta$ | $\cos\theta$ | $\cos\theta$ |
| tan | $\cot\theta$ | $-\cot\theta$ | $-\tan\theta$ | $\tan\theta$ | $\cot\theta$ | $-\cot\theta$ | $-\tan\theta$ | $\tan\theta$ |
| cot | $\tan\theta$ | $-\tan\theta$ | $-\cot\theta$ | $\cot\theta$ | $\tan\theta$ | $-\tan\theta$ | $-\cot\theta$ | $\cot\theta$ |
| sec | $\text{cosec}\theta$ | $-\text{cosec}\theta$ | $-\sec\theta$ | $-\sec\theta$ | $-\text{cosec}\theta$ | $\text{cosec}\theta$ | $\sec\theta$ | $\sec\theta$ |
| cosec | $\sec\theta$ | $\sec\theta$ | $\text{cosec}\theta$ | $-\text{cosec}\theta$ | $-\sec\theta$ | $-\sec\theta$ | $-\text{cosec}\theta$ | $\text{cosec}\theta$ |

## 05. Recapitulation of previous class

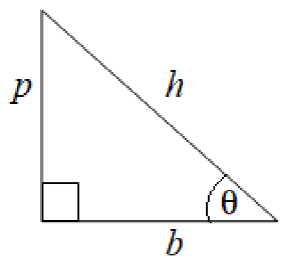
Following is a list of those relations which you have studied in your last class.
Please note that their proof has not been mentioned here. Though you can anytime discuss it with your teacher again in case you have forgotten!

(a) *Trigonometric ratios and sides of a right angled triangle:*

(i) $\sin\theta=\frac{p}{h}$ (ii) $\cos\theta=\frac{b}{h}$

(iii) $\text{cosec}\,\theta=\frac{h}{p}$ (iv) $\tan\theta=\frac{p}{b}$

(v) $\sec\theta=\frac{h}{b}$ (vi) $\cot\theta=\frac{b}{p}$




(b) *Trigonometric Identities:*

(i) $\sin^2\theta+\cos^2\theta=1$

Also $\cos\theta=\pm\sqrt{1-\sin^2\theta}$, $\sin\theta=\pm\sqrt{1-\cos^2\theta}$.

(ii) $1+\tan^2\theta=\sec^2\theta$

Also $\sec\theta=\pm\sqrt{1+\tan^2\theta}$, $\tan\theta=\pm\sqrt{\sec^2\theta-1}$.

(iii) $1+\cot^2\theta=\text{cosec}^2\theta$

Also $\text{cosec}\theta=\pm\sqrt{1+\cot^2\theta}$, $\cot\theta=\pm\sqrt{\text{cosec}^2\theta-1}$.

(c) *Relation between trigonometric ratios:*

(i) $\tan\theta=\frac{\sin\theta}{\cos\theta}$ (ii) $\tan\theta=\frac{1}{\cot\theta}$

(iii) $\tan\theta.\cot\theta=1$ (iv) $\cot\theta=\frac{\cos\theta}{\sin\theta}$

(v) $\text{cosec}\,\theta=\frac{1}{\sin\theta}$ (vi) $\sec\theta=\frac{1}{\cos\theta}$.

☑ Following table includes trigonometric ratio of standard angles:

| Degree / Radian $(\rightarrow)$ | $0^\circ$ | $30^\circ$ | $45^\circ$ | $60^\circ$ | $90^\circ$ |
|---|---|---|---|---|---|
| T-Ratios $(\downarrow)$ | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ |
| sin | 0 | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| cos | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 |
| tan | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | $\infty$ |
| cosec | $\infty$ | 2 | $\sqrt{2}$ | $\frac{2}{\sqrt{3}}$ | 1 |
| sec | 1 | $\frac{2}{\sqrt{3}}$ | $\sqrt{2}$ | 2 | $\infty$ |
| cot | $\infty$ | $\sqrt{3}$ | 1 | $\frac{1}{\sqrt{3}}$ | 0 |

☑ Following table demonstrates the **Domain** and **Range** of trigonometric functions:

| T-Functions $(\downarrow)$ | Domain | Range |
|---|---|---|
| sin x | R | $[-1, 1]$ |
| cos x | R | $[-1, 1]$ |
| tan x | $\left\{x \in R : x \neq (2n \pm 1)\frac{\pi}{2}, n \in Z\right\}$ | R |
| cot x | $\{x \in R : x \neq n\pi, n \in Z\}$ | R |
| cosec x | $\{x \in R : x \neq n\pi, n \in Z\}$ | $R - (-1, 1)$ |
| sec x | $\left\{x \in R : x \neq (2n \pm 1)\frac{\pi}{2}, n \in Z\right\}$ | $R - (-1, 1)$ |

**Graphs** of Trigonometric Functions should be discussed in the class.
*At the end of this chapter, graphs have been given for all T-functions.*

## 06. Some useful trigonometric identities and formulae

☑ **Trigonometric identities**

**(a)** $\sin^2\theta + \cos^2\theta = 1$

**(b)** $1 + \tan^2\theta = \sec^2\theta$

**(c)** $1 + \cot^2\theta = \text{cosec}^2\theta$

☑ **Addition / subtraction formulae & some related results**

**(a)** $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

**(b)** $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

**(c)** $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

**(d)** $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

**(e)** $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

**(f)** $\cot(A \pm B) = \dfrac{\cot B \cot A \mp 1}{\cot B \pm \cot A}$

☑ **Transformation of sums / differences into products & vice-versa**

**(a)** $\sin C + \sin D = 2\sin\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

**(b)** $\sin C - \sin D = 2\cos\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

**(c)** $\cos C + \cos D = 2\cos\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

**(d)** $\cos C - \cos D = -2\sin\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

**(e)** $2\sin A \cos B = \sin(A+B) + \sin(A-B)$

**(f)** $2\cos A \sin B = \sin(A+B) - \sin(A-B)$

**(g)** $2\cos A \cos B = \cos(A+B) + \cos(A-B)$

**(h)** $2\sin A \sin B = \cos(A-B) - \cos(A+B)$

☑ **Multiple angle formulae involving 2A & 3A**

**(a)** $\sin 2A = 2\sin A \cos A$

**(b)** $\sin A = 2\sin\dfrac{A}{2}\cos\dfrac{A}{2}$

**(c)** $\cos 2A = \cos^2 A - \sin^2 A$

**(d)** $\cos A = \cos^2\dfrac{A}{2} - \sin^2\dfrac{A}{2}$

**(e)** $\cos 2A = 2\cos^2 A - 1$

**(f)** $2\cos^2 A = 1 + \cos 2A$

**(g)** $\cos 2A = 1 - 2\sin^2 A$

**(h)** $2\sin^2 A = 1 - \cos 2A$

**(i)** $\sin 2A = \dfrac{2\tan A}{1+\tan^2 A}$

**(j)** $\cos 2A = \dfrac{1-\tan^2 A}{1+\tan^2 A}$

**(k)** $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

**(l)** $\sin 3A = 3\sin A - 4\sin^3 A$

**(m)** $\cos 3A = 4\cos^3 A - 3\cos A$

**(n)** $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

# WORKED OUT ILLUSTRATIVE EXAMPLES

**Ex01.** Find the radian measure corresponding to (a) $240^\circ$ (b) $-37^\circ 30'$.

**Sol.** We know that, $\text{Angle in Radian Measure} = (\text{Angle in Degree Measure}) \times \dfrac{\pi}{180}$

(a) Radian Measure of $240^\circ = \left(240 \times \dfrac{\pi}{180}\right)^c = \left(\dfrac{4\pi}{3}\right)^c$ or simply, $\dfrac{4\pi}{3}$.

(b) As $-37^\circ 30' = -\left(37\dfrac{1}{2}\right)^\circ = -\left(\dfrac{75}{2}\right)^\circ$

So, the Radian Measure of $\left(-\dfrac{75}{2}\right)^\circ = \left[\left(-\dfrac{75}{2}\right) \times \dfrac{\pi}{180}\right]^c = \left(-\dfrac{5\pi}{24}\right)^c$ or simply, $-\dfrac{5\pi}{24}$.

**Ex02.** Find the degree measure corresponding to (a) $-2$ (b) $\left(\frac{7\pi}{12}\right)^c$.

**Sol.** We know that, $\textit{Angle in Degree Measure} = (\textit{Angle in Radian Measure}) \times \frac{180}{\pi}$

(a) Degree Measure of $-2 = \left(-2 \times \frac{180}{\pi}\right)^o = \left(-2 \times \frac{180}{22/7}\right)^o = -(114^o32'44'')$.

(b) Degree Measure of $\left(\frac{7\pi}{12}\right)^c = \left(\frac{7\pi}{12} \times \frac{180}{\pi}\right)^o = 105^o$.

**Ex03.** The minute hand of a watch is known to be 1.4 cm long.
How far does its tip move in 45 minutes? Use $\pi = \frac{22}{7}$.

**Sol.** In 60 minutes, the minute hand moves through $2\pi$ radians.

So, in 45 minutes, it moves through $\left(\frac{2\pi}{60} \times 45\right)^c = \left(\frac{3\pi}{2}\right)^c$.

Now by using $\theta\,(\text{in radians}) = \frac{l}{r}$, we have $l = 1.4 \times \frac{3\pi}{2}$ cm

$\Rightarrow \quad = 1.4 \times \frac{3}{2} \times \frac{22}{7} = 6.6$ cm.

Hence, the distance moved by the tip of the minute hand in 45 minutes is 6.6 cm.

**Ex04.** A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

**Sol.** No. of revolutions done in one minute (60 seconds) $= 360$

No. of revolutions done in one second $= \frac{360}{60} = 6$

Now, angle made in one revolution $= 2\pi$ radians

Therefore, the angle made in 6 revolutions $= 12\pi$ radians.

**Ex05.** If $\sin\theta = -\frac{4}{5}$, $\pi < \theta < \frac{3\pi}{2}$, then find the value of $\tan\theta$, $\cos\theta$ and $\sec\theta$.

**Sol.** It is clearly evident that $\theta$ lies in third quadrant in which only $\tan\theta$ and $\cot\theta$ are positive and all the remaining trigonometric functions are negative.

Now, $\sin\theta = -\frac{4}{5}$

$\because \quad \cos\theta = \pm\sqrt{1-\sin^2\theta}$

$\therefore \cos\theta = -\sqrt{1-\left(-\frac{4}{5}\right)^2}$

$\Rightarrow \cos\theta = -\frac{3}{5}$ (As $\theta$ lies in 3rd quadrant)

So, $\sec\theta = \frac{1}{\cos\theta}$

$\Rightarrow \sec\theta = -\frac{5}{3}$.

And $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\Rightarrow\ \tan\theta = \dfrac{-4/5}{-3/5}$

$\Rightarrow\ \tan\theta = \dfrac{4}{3}$.

**Ex06.** Evaluate: (a) $\operatorname{cosec}\left(-1410^\circ\right)$ (b) $\cos\left(-870^\circ\right)$.

**Sol.** (a) $\operatorname{cosec}\left(-1410^\circ\right) = \operatorname{cosec}\left(-1410^\circ + 4\times 360^\circ\right)$

$\Rightarrow\ = \operatorname{cosec}\left(-1410^\circ + 1440^\circ\right) = \operatorname{cosec}30^\circ = 2$.

(b) $\cos\left(-870^\circ\right) = \cos\left(-870^\circ + 3\times 360^\circ\right) = \cos\left(-870^\circ + 1080^\circ\right) = \cos 210^\circ$.

$\Rightarrow\ = \cos\left(180^\circ + 30^\circ\right) = -\cos 30^\circ = -\dfrac{\sqrt{3}}{2}$.

**Ex07.** Evaluate: (a) $\tan\dfrac{5\pi}{12}$ (b) $\sin 18^\circ$.

**Sol.** (a) $\tan\dfrac{5\pi}{12} = \tan\left(\dfrac{\pi}{4} + \dfrac{\pi}{6}\right) = \dfrac{\tan\frac{\pi}{4} + \tan\frac{\pi}{6}}{1 - \tan\frac{\pi}{4}\tan\frac{\pi}{6}} = \dfrac{1 + \frac{1}{\sqrt{3}}}{1 - 1.\frac{1}{\sqrt{3}}}$

$\Rightarrow \tan\dfrac{5\pi}{12} = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$

$\therefore\ \tan\dfrac{5\pi}{12} = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$.

(b) Let $\theta = 18^\circ$

$\Rightarrow 5\theta = 90^\circ$

$\Rightarrow 2\theta = 90^\circ - 3\theta$

$\Rightarrow \sin 2\theta = \sin\left(90^\circ - 3\theta\right)$

$\Rightarrow 2\sin\theta\cos\theta = \cos 3\theta$

$\Rightarrow 2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$

$\Rightarrow \cos\theta\left(2\sin\theta - 4\cos^2\theta + 3\right) = 0$ $\left[\because\ \cos\theta = \cos 18^\circ \neq 0\right.$

$\Rightarrow 2\sin\theta - 4\left(1 - \sin^2\theta\right) + 3 = 0$

$\Rightarrow 4\sin^2\theta + 2\sin\theta - 1 = 0$

$\Rightarrow \sin\theta = \dfrac{-2 \pm \sqrt{4+16}}{8} = \dfrac{-1 \pm \sqrt{5}}{4}$

$\Rightarrow \sin 18^\circ = \dfrac{\sqrt{5}-1}{4}$. $\left[\because\ \sin 18^\circ > 0\right.$

**Ex08.** Prove that: $\sin 15^\circ - \cos 15^\circ = -\dfrac{1}{\sqrt{2}}$.

**Sol.** **LHS :** $\sin 15^\circ - \cos 15^\circ$

$\Rightarrow\ = \sin 15^\circ - \cos(90^\circ - 75^\circ)$

$\Rightarrow\ = \sin 15^\circ - \sin 75^\circ$

$\Rightarrow\ = 2\cos\dfrac{15^\circ + 75^\circ}{2}\sin\dfrac{15^\circ - 75^\circ}{2}$

$\Rightarrow\ = 2\cos 45^\circ \sin(-30^\circ)$

$\Rightarrow \quad = 2\times\frac{1}{\sqrt{2}}\times\{-\sin 30^o\}$

$\Rightarrow \quad = -2\times\frac{1}{\sqrt{2}}\times\frac{1}{2}$

$\Rightarrow \quad = -\frac{1}{\sqrt{2}} =$ **RHS.**

**Ex09.** Prove that: $\tan 75^o + \cot 75^o = 4$.

**Sol.** **LHS :** $\tan 75^o + \cot 75^o$

$\Rightarrow \quad = \frac{\sin 75^o}{\cos 75^o} + \frac{\cos 75^o}{\sin 75^o}$

$\Rightarrow \quad = \frac{\sin^2 75^o + \cos^2 75^o}{\sin 75^o \cos 75^o}$

$\Rightarrow \quad = \frac{1}{2\sin 75^o \cos 75^o}\times 2$ (Using $\sin^2 x + \cos^2 x = 1$

$\Rightarrow \quad = \frac{2}{\sin 150^o} = \frac{2}{\sin(180^o - 30^o)} = \frac{2}{\sin 30^o} = \frac{2}{\frac{1}{2}}$

$\Rightarrow \quad = 4 =$ **RHS**.

**Ex10.** Show that $\frac{1+\sin\theta+\cos\theta}{1+\sin\theta-\cos\theta} = \cot\frac{\theta}{2}$.

**Sol.** **LHS :** $\frac{1+\sin\theta+\cos\theta}{1+\sin\theta-\cos\theta}$

$\Rightarrow \quad = \frac{(1+\cos\theta)+\sin\theta}{(1-\cos\theta)+\sin\theta}$

$\Rightarrow \quad = \frac{2\cos^2\frac{\theta}{2} + 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2} + 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$

$\Rightarrow \quad = \frac{2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)}{2\sin\frac{\theta}{2}\left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right)}$

$\Rightarrow \quad = \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}$

$\Rightarrow \quad = \cot\frac{\theta}{2} =$ **RHS**.

**Ex11.** Show that $\frac{\cos x - \sin x}{\cos x + \sin x} + \frac{\cos x + \sin x}{\cos x - \sin x} = 2\sec 2x$.

**Sol.** **LHS :** $\frac{\cos x - \sin x}{\cos x + \sin x} + \frac{\cos x + \sin x}{\cos x - \sin x}$

$\Rightarrow \quad = \frac{(\cos x - \sin x)^2 + (\cos x + \sin x)^2}{(\cos x + \sin x)(\cos x - \sin x)}$

$\Rightarrow \quad = \dfrac{(\cos^2 x - 2\cos x \sin x + \sin^2 x) + (\cos^2 x + 2\cos x \sin x + \sin^2 x)}{(\cos^2 x - \sin^2 x)}$

$\Rightarrow \quad = \dfrac{2}{\cos 2x}$

$\Rightarrow \quad = 2\sec 2x =$ **RHS.**

**Ex12.** Prove that $\cos x \cos\dfrac{x}{4} - \cos\dfrac{3x}{2}\cos\dfrac{9x}{4} = \sin\dfrac{5x}{2}\sin\dfrac{5x}{4}$.

**Sol.** **LHS :** $\cos x \cos\dfrac{x}{4} - \cos\dfrac{3x}{2}\cos\dfrac{9x}{4}$

$\Rightarrow \quad = \dfrac{1}{2}\left[2\cos x \cos\dfrac{x}{4} - 2\cos\dfrac{3x}{2}\cos\dfrac{9x}{4}\right]$

$\Rightarrow \quad = \dfrac{1}{2}\left[\left\{\cos\left(x+\dfrac{x}{4}\right) + \cos\left(x-\dfrac{x}{4}\right)\right\} - \left\{\cos\left(\dfrac{3x}{2}+\dfrac{9x}{4}\right) + \cos\left(\dfrac{3x}{2}-\dfrac{9x}{4}\right)\right\}\right]$

$\Rightarrow \quad = \dfrac{1}{2}\left[\left\{\cos\dfrac{5x}{4} + \cos\dfrac{3x}{4}\right\} - \left\{\cos\dfrac{15x}{4} + \cos\left(-\dfrac{3x}{4}\right)\right\}\right]$ $\quad \{\because \cos(-x) = \cos x$

$\Rightarrow \quad = \dfrac{1}{2}\left[\left\{\cos\dfrac{5x}{4} + \cos\dfrac{3x}{4}\right\} - \left\{\cos\dfrac{15x}{4} + \cos\dfrac{3x}{4}\right\}\right]$

$\Rightarrow \quad = \dfrac{1}{2}\left[\cos\dfrac{5x}{4} - \cos\dfrac{15x}{4}\right]$

$\Rightarrow \quad = \dfrac{1}{2}\left[-2\sin\dfrac{1}{2}\left(\dfrac{5x}{4}+\dfrac{15x}{4}\right)\sin\dfrac{1}{2}\left(\dfrac{5x}{4}-\dfrac{15x}{4}\right)\right]$

$\Rightarrow \quad = \dfrac{1}{2}\left[-2\sin\dfrac{5x}{2}\sin\left(-\dfrac{5x}{4}\right)\right]$ $\quad \{\because \sin(-x) = -\sin x$

$\Rightarrow \quad = \dfrac{1}{2}\left[2\sin\dfrac{5x}{2}\sin\dfrac{5x}{4}\right]$

$\Rightarrow \quad = \sin\dfrac{5x}{2}\sin\dfrac{5x}{4} =$ **RHS.**

**Ex13.** Prove that: $\sin 2x \sin\dfrac{x}{2} - \sin 3x \sin\dfrac{9x}{2} = -\sin 5x \sin\dfrac{5x}{2}$.

**Sol.** **LHS :** $\sin 2x \sin\dfrac{x}{2} - \sin 3x \sin\dfrac{9x}{2}$

$\Rightarrow \quad = \dfrac{1}{2}\left[2\sin 2x \sin\dfrac{x}{2} - 2\sin 3x \sin\dfrac{9x}{2}\right]$

$\Rightarrow \quad = \dfrac{1}{2}\left[\left\{\cos\left(2x-\dfrac{x}{2}\right) - \cos\left(2x+\dfrac{x}{2}\right)\right\} - \left\{\cos\left(3x-\dfrac{9x}{2}\right) - \cos\left(3x+\dfrac{9x}{2}\right)\right\}\right]$

$\Rightarrow \quad = \dfrac{1}{2}\left[\left\{\cos\dfrac{3x}{2} - \cos\dfrac{5x}{2}\right\} - \left\{\cos\left(\dfrac{-3x}{2}\right) - \cos\dfrac{15x}{2}\right\}\right]$

$\Rightarrow \quad = \dfrac{1}{2}\left[\left\{\cos\dfrac{3x}{2} - \cos\dfrac{5x}{2}\right\} - \left\{\cos\dfrac{3x}{2} - \cos\dfrac{15x}{2}\right\}\right]$

$\Rightarrow \quad = \dfrac{1}{2}\left[\cos\dfrac{15x}{2} - \cos\dfrac{5x}{2}\right]$

$\Rightarrow \quad = \frac{1}{2}\left[-2\sin\frac{1}{2}\left(\frac{15x}{2}+\frac{5x}{2}\right)\sin\frac{1}{2}\left(\frac{15x}{2}-\frac{5x}{2}\right)\right]$

$\Rightarrow \quad = -\sin 5x \sin\frac{5x}{2} = \textbf{RHS}.$

**Ex14.** Prove that: $\sin(40^\circ+\theta)\cos(10^\circ+\theta)-\cos(40^\circ+\theta)\sin(10^\circ+\theta)=\frac{1}{2}$.

**Sol.** We know that $\sin A\cos B-\cos A\sin B=\sin(A-B)$.

Putting $A=40^\circ+\theta$ and $B=10^\circ+\theta$ in this identity, we get

$\sin(40^\circ+\theta)\cos(10^\circ+\theta)-\cos(40^\circ+\theta)\sin(10^\circ+\theta)$

$\Rightarrow \quad = \sin\left[(40^\circ+\theta)-(10^\circ+\theta)\right]$

$\Rightarrow \quad = \sin 30^\circ = \frac{1}{2}.$

That is, $\sin(40^\circ+\theta)\cos(10^\circ+\theta)-\cos(40^\circ+\theta)\sin(10^\circ+\theta)=\frac{1}{2}$.

**Ex15.** Prove that $\cos x+\cos 3x+\cos 5x+\cos 7x = 4\cos 4x\cos 2x\cos x$.

**Sol.** **LHS :** $\cos x+\cos 3x+\cos 5x+\cos 7x$

$\Rightarrow \quad = (\cos x+\cos 7x)+(\cos 3x+\cos 5x)$

$\Rightarrow \quad = 2\cos\frac{x+7x}{2}\cos\frac{x-7x}{2}+2\cos\frac{3x+5x}{2}\cos\frac{3x-5x}{2}$

$\Rightarrow \quad = 2\cos 4x\cos(-3x)+2\cos 4x\cos(-x)$

$\Rightarrow \quad = 2\cos 4x\left[\cos 3x+\cos x\right]$

$\Rightarrow \quad = 2\cos 4x\left[2\cos\frac{3x+x}{2}\cos\frac{3x-x}{2}\right]$

$\Rightarrow \quad = 2\cos 4x\left[2\cos 2x\cos x\right]$

$\Rightarrow \quad = 4\cos 4x\cos 2x\cos x = \textbf{RHS}.$

**Ex16.** Prove that $(\cos x-\cos y)^2-(\sin x-\sin y)^2 = -4\sin^2\left(\frac{x-y}{2}\right)\cos(x+y)$.

**Sol.** **LHS :** $(\cos x-\cos y)^2-(\sin x-\sin y)^2$

$\Rightarrow \quad = \cos^2 x-2\cos x\cos y+\cos^2 y-\sin^2 x+2\sin x\sin y-\sin^2 y$

$\Rightarrow \quad = (\cos^2 x-\sin^2 x)-2(\cos x\cos y-\sin x\sin y)+(\cos^2 y-\sin^2 y)$

$\Rightarrow \quad = \cos 2x-2\cos(x+y)+\cos 2y$

$\Rightarrow \quad = \cos 2x+\cos 2y-2\cos(x+y)$

$\Rightarrow \quad = 2\cos\frac{2x+2y}{2}\cos\frac{2x-2y}{2}-2\cos(x+y)$

$\Rightarrow \quad = 2\cos(x+y)\cos(x-y)-2\cos(x+y)$

$\Rightarrow \quad = 2\cos(x+y)\left[\cos(x-y)-1\right]$

$\Rightarrow \quad = -2\cos(x+y)\left[1-\cos(x-y)\right]$

$\Rightarrow \quad = -2\cos(x+y)\left[2\sin^2\frac{x-y}{2}\right]$

$\Rightarrow \quad = -4\cos(x+y)\sin^2\left(\frac{x-y}{2}\right) = \textbf{RHS}.$

**Ex17.** Prove that $\sin 20^\circ \sin 40^\circ \sin 80^\circ = \frac{\sqrt{3}}{8}$.

**Sol.** **LHS :** $\sin 20^\circ \sin 40^\circ \sin 80^\circ$

$\Rightarrow \quad = \frac{1}{2} \times (2\sin 20^\circ \sin 40^\circ) \sin 80^\circ$

$\Rightarrow \quad = \frac{1}{2} \times \left[\cos(20^\circ - 40^\circ) - \cos(20^\circ + 40^\circ)\right] \sin 80^\circ$

$\Rightarrow \quad = \frac{1}{2} \times \left[\cos(-20^\circ) - \cos 60^\circ\right] \sin 80^\circ$

$\Rightarrow \quad = \frac{1}{2} \times \left[\sin 80^\circ \cos 20^\circ - \sin 80^\circ \cos 60^\circ\right]$

$\Rightarrow \quad = \frac{1}{4} \times \left[2\sin 80^\circ \cos 20^\circ - 2\sin 80^\circ \times \frac{1}{2}\right]$

$\Rightarrow \quad = \frac{1}{4} \times \left[\left\{\sin(80^\circ + 20^\circ) + \sin(80^\circ - 20^\circ)\right\} - \sin 80^\circ\right]$

$\Rightarrow \quad = \frac{1}{4} \times \left[\left\{\sin 100^\circ + \sin 60^\circ\right\} - \sin 80^\circ\right]$

$\Rightarrow \quad = \frac{1}{4} \times \left[\left\{\sin(180^\circ - 80^\circ) + \frac{\sqrt{3}}{2}\right\} - \sin 80^\circ\right]$

$\Rightarrow \quad = \frac{1}{4} \times \left[\sin 80^\circ + \frac{\sqrt{3}}{2} - \sin 80^\circ\right]$

$\Rightarrow \quad = \frac{\sqrt{3}}{8} =$ **RHS**.

**Ex18.** If $\cos x = -\frac{1}{3}$ and $x$ lies in III quadrant, find the values of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$.

**Sol.** Since $x$ lies in III quadrant, we have $\pi < x < \frac{3\pi}{2}$ which implies, $\frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$ i.e., $\frac{x}{2}$ lies in the II quadrant.

$\therefore \ \sin\frac{x}{2} > 0$, $\cos\frac{x}{2} < 0$ and $\tan\frac{x}{2} < 0$.

Now as we know, $2\sin^2\frac{x}{2} = (1 - \cos x)$

$\Rightarrow 2\sin^2\frac{x}{2} = 1 + \frac{1}{3} = \frac{4}{3}$

$\Rightarrow \sin^2\frac{x}{2} = \frac{2}{3}$

$\Rightarrow \sin\frac{x}{2} = \pm\sqrt{\frac{2}{3}}$

$\therefore \ \sin\frac{x}{2} = \sqrt{\frac{2}{3}}$ $\qquad \left[\because \frac{x}{2} \in \text{II Quadrant}\right.$

$\Rightarrow \sin\frac{x}{2} = \frac{\sqrt{6}}{3}$.

Also $2\cos^2\frac{x}{2}=\left(1+\cos x\right)$

$\Rightarrow 2\cos^2\frac{x}{2}=1-\frac{1}{3}=\frac{2}{3}$

$\Rightarrow \cos^2\frac{x}{2}=\frac{1}{3}$

$\Rightarrow \cos\frac{x}{2}=-\frac{1}{\sqrt{3}}$ *i.e.,* $\cos\frac{x}{2}=-\frac{\sqrt{3}}{3}$.

Therefore, by using $\tan\frac{x}{2}=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}$ we get,

$\therefore\ \tan\frac{x}{2}=\frac{\frac{\sqrt{6}}{3}}{-\frac{\sqrt{3}}{3}}$ *i.e.,* $\tan\frac{x}{2}=-\sqrt{2}$.

**Ex19.** If $\cot\alpha\cot\beta=2$, show that $\frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}=\frac{1}{3}$.

**Sol.** As $\cot\alpha\cot\beta=2$

$\Rightarrow \frac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta}=\frac{2}{1}$

$\Rightarrow \frac{\cos\alpha\cos\beta+\sin\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}=\frac{2+1}{2-1}$ (Using Componendo & Dividendo

$\Rightarrow \frac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)}=\frac{3}{1}$

$\therefore\ \frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}=\frac{1}{3}$.

**Ex20.** Find the value of $\cot\frac{\pi}{24}$.

**Sol.** Let $y=\cot\frac{\pi}{24}=\frac{\cos\frac{\pi}{24}}{\sin\frac{\pi}{24}}$

$\Rightarrow y=\frac{2\cos\frac{\pi}{24}\cos\frac{\pi}{24}}{2\sin\frac{\pi}{24}\cos\frac{\pi}{24}}$

$\Rightarrow y=\frac{2\cos^2\frac{\pi}{24}}{\sin\frac{\pi}{12}}$

$\Rightarrow y=\frac{1+\cos\frac{\pi}{12}}{\sin\frac{\pi}{12}}$

$$\Rightarrow y=\frac{1+\cos\left(\frac{\pi}{4}-\frac{\pi}{6}\right)}{\sin\left(\frac{\pi}{4}-\frac{\pi}{6}\right)}$$

$$\Rightarrow y=\frac{1+\cos\frac{\pi}{4}\cos\frac{\pi}{6}+\sin\frac{\pi}{4}\sin\frac{\pi}{6}}{\sin\frac{\pi}{4}\cos\frac{\pi}{6}-\cos\frac{\pi}{4}\sin\frac{\pi}{6}}$$

$$\Rightarrow y=\frac{1+\frac{\sqrt{3}}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}}{\frac{\sqrt{3}}{2\sqrt{2}}-\frac{1}{2\sqrt{2}}}$$

$$\Rightarrow y=\frac{\frac{2\sqrt{2}+\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}}$$

$$\Rightarrow y=\frac{2\sqrt{2}+\sqrt{3}+1}{\sqrt{3}-1}$$

$$\Rightarrow y=\frac{2\sqrt{2}+\sqrt{3}+1}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$\Rightarrow y=\frac{2\sqrt{6}+2\sqrt{2}+2\sqrt{3}+4}{2}$$

$$\Rightarrow y=\sqrt{2}+\sqrt{3}+2+\sqrt{6}$$

$$\therefore y=\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}.$$

# Exercise 3.1

## Basic Introductive Questions

**Q01**. Find the radian measures corresponding to the following degree measures:

(a) $250^\circ$ (b) $-47^\circ 30'$ (c) $25^\circ$

(d) $45^\circ 20' 10''$ (e) $81^\circ$.

**Q02**. Find the degree measures corresponding to the following radian measures:

(a) $\left(\frac{11}{16}\right)^c$ (b) $-4$ (c) 6

(d) $\frac{7\pi}{6}$ (e) $(1.2)^c$.

**Q03**. Find degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm.

**Q04**. In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.

**Q05**. If the arcs of same lengths in two circles subtend angles $65^\circ$ and $110^\circ$ at the centre, then find the ratio of their radii.

**Q06**. The minute hand of a watch is 1.5 cm long. How far does its tip move in 40 minutes?

**Q07**. The angles of a triangle are in A.P. and the ratio of the number of degrees in the least to the number of radians in the greatest is $60:\pi$. Find the angles in degrees and radians.

**Q08**. The difference between the two acute angles of a right-angled triangle is $\frac{2}{5}\pi$. Find the angles in degrees and in the radians.

**Q09**. In a right-angled triangle, the difference between the two acute angles is $\left(\frac{\pi}{15}\right)^c$. Find the angles in degrees.

**Q10**. A train is moving on a circular curve of radius 1500 m at the rate of 66 km/hr. Through what angle has it turned in 10 seconds?

**Q11.** Find the length which at a distance of 5280 m will subtend an angle of $1'$ at the eye.

**Q12.** Find each interior angle of a regular decagon in radian.

**Q13.** Find the angle between hour hand and minute hand of a clock at 7:20.

**Q14.** Find the angle between hour hand and minute hand of a clock at quarter to five.

**Q15.** If the angles of a triangle are in the ratio 3:4:5, then find the smallest angle in the radian and the greatest in the degree.

**Q16.** A horse is tied to a post by a rope. If the horse moves along a circular path, always keeping the rope tight and describes 88 m. When it traces $72^\circ$ at the centre, find the length of the rope.

**Q17**. If $\sin x = -\frac{2\sqrt{6}}{5}$ and x lies in III quadrant, find the values of other five trigonometric functions.

**Q18**. If $\cot x = -\frac{5}{12}$, x lies in second quadrant, find the values of other five trigonometric functions.

**Q19**. Evaluate the followings:

(a) $\sin\frac{25\pi}{3}$ (b) $\tan\left(-\frac{16\pi}{3}\right)$ (c) $\text{cosec}\left(-\frac{33\pi}{4}\right)$ (d) $\sec\left(\frac{19}{3}\pi\right)$.

**Q20**. Find the value of (a) $\text{cosec}(-1110^\circ)$ (b) $\sin 765^\circ$.

**Q21.** Evaluate : $\sin(-1470^\circ)+\sqrt{3}\cos(-1470^\circ)$.

**Q22.** If $\sec x = \sqrt{2}$ and $\frac{3\pi}{2} < x < 2\pi$, then find the value of $\frac{1+\tan x+\text{cosec } x}{1+\cot x-\text{cosec } x}$.

**Q23.** Convert $\frac{1}{4}$ into degrees.

## Exercise 3.2
### Concept Building Questions - I

**Q01**. Find the value of followings:

(a) $\sin 15^\circ$ (b) $\cos 75^\circ$ (c) $\tan 15^\circ$

(d) $\tan 75^\circ$ (e) $\sin 105^\circ$ (f) $\tan 105^\circ$.

**Q02**. Find the value of the followings:

(a) $\tan\frac{13\pi}{12}$ (b) $\tan\frac{\pi}{8}$

(c) $\sin\frac{\pi}{12}$ (d) $\sin\frac{7\pi}{8}$.

**Q03.** Express $2\cos 4x \sin 2x$ as an algebraic sum of sines and/or cosines.

**Q04.** Express $\sin 12\theta + \sin 4\theta$ as the product of sines and cosines.

**Q05**. If $\sin x=\frac{3}{5}$, $\cos y=-\frac{12}{13}$ where x and y both lie in the second quadrant, then find the values of $\sin(x+y)$, $\cos(x+y)$ and $\tan(x-y)$.

**Q06**. If $\sin\theta=\frac{15}{17}$, $\cos\phi=\frac{12}{13}$ where $\theta$ and $\phi$ both lie in the first quadrant, then determine the values of $\sin(\theta+\phi)$, $\cos(\theta-\phi)$ and $\tan(\theta+\phi)$.

**Q07**. If $\cot\alpha=\frac{1}{2}$, $\sec\beta=-\frac{5}{3}$ where $\alpha\in\left(\pi,\frac{3\pi}{2}\right)$ and $\beta\in\left(\frac{\pi}{2},\pi\right)$, then find the value of $\tan(\alpha+\beta)$.

**Q08**. If $\sec x=-\frac{13}{12}$, $\frac{\pi}{2}<x<\pi$, find the value of $\sin 2x$ and $\tan 2x$.

**Q09**. If $\tan x=-\frac{4}{3}$, where x lie in II Quadrant, find the value of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$.

**Q10**. Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ in each of the followings:

(a) $\cos x=-\frac{1}{3}$, $x\in$ II quadrant (b) $\sin x=-\frac{1}{4}$, $\pi<x<\frac{3\pi}{2}$ (c) $\tan x=-\frac{3}{4}$, $90^\circ<x<180^\circ$.

**Q11**. Evaluate: $\frac{\sin 75^\circ-\sin 15^\circ}{\cos 75^\circ+\cos 15^\circ}$.

**Q12**. If $\tan A=\frac{a}{a+1}$ and $\tan B=\frac{1}{2a+1}$, then find the value of $A+B$.

## Exercise 3.3
## Concept Building Questions - II

**Q01**. Prove the followings:

**Level - I**

(a) $\sin 105^\circ+\cos 105^\circ=\frac{1}{\sqrt{2}}$

(b) $\cos 15^\circ-\sin 15^\circ=\frac{1}{\sqrt{2}}$

(c) $\cos 18^\circ-\sin 18^\circ=\sqrt{2}\sin 27^\circ$

(d) $\cos 9^\circ+\sin 9^\circ=\sqrt{2}\sin 54^\circ$

(e) $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)=-\sqrt{2}\sin x$

(f) $\sin^2 5^\circ+\sin^2 10^\circ+\sin^2 15^\circ+...+\sin^2 90^\circ=9\frac{1}{2}$

(g) $\frac{\cos 11^\circ+\sin 11^\circ}{\cos 11^\circ-\sin 11^\circ}=\tan 56^\circ$

(h) $\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}=\tan\left(\frac{\theta}{2}\right)$

(i) $\frac{\sec 8x-1}{\sec 4x-1}=\frac{\tan 8x}{\tan 2x}$

(j) $\frac{\tan 5\theta+\tan 3\theta}{\tan 5\theta-\tan 3\theta}=4\cos 2\theta\cos 4\theta$

(k) $\frac{\cos x}{1-\sin x}=\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$

(l) $\frac{\sin(B-C)}{\cos B\cos C}+\frac{\sin(C-A)}{\cos C\cos A}+\frac{\sin(A-B)}{\cos A\cos B}=0$

(m) $\sqrt{2-\sqrt{2+2\cos 4\theta}}=2\sin\theta$, $0<\theta<\frac{\pi}{4}$

(n) $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8x}}}=2\cos x$, $x\in\left(0,\frac{\pi}{8}\right)$

(o) $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)=\frac{1}{8}$

**Level - II**

(a) $\tan 40^\circ+2\tan 10^\circ=\tan 50^\circ$

(b) $\tan 70^\circ=\tan 20^\circ+2\tan 50^\circ$

(c) $\sin^2\left(\frac{\pi}{8}+\frac{x}{2}\right)-\sin^2\left(\frac{\pi}{8}-\frac{x}{2}\right)=\frac{1}{\sqrt{2}}\sin x$ (d) $\sin^2 6x-\sin^2 4x=\sin 2x\sin 10x$

(e) $\sin 2x+2\sin 4x+\sin 6x=4\cos^2 x\sin 4x$ (f) $\sin 3x+\sin 2x-\sin x=4\sin x\cos\frac{x}{2}\cos\frac{3x}{2}$

(g) $\sin x+\sin 3x+\sin 5x+\sin 7x=4\cos x\cos 2x\sin 4x$

(h) $\cos\alpha+\cos\beta+\cos\gamma+\cos(\alpha+\beta+\gamma)=4\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\beta+\gamma}{2}\right)\cos\left(\frac{\gamma+\alpha}{2}\right)$

(i) $\cos\left(\frac{\pi}{4}-\theta\right)\cos\left(\frac{\pi}{4}-\phi\right)-\sin\left(\frac{\pi}{4}-\theta\right)\sin\left(\frac{\pi}{4}-\phi\right)=\sin(\theta+\phi)$

(j) $\sin(n+1)x\sin(n+2)x+\cos(n+1)x\cos(n+2)x=\cos x$

(k) $\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]=1$

(l) $\frac{\cos(90^\circ+x)\sec(270^\circ+x)\sin(180^\circ+x)}{\operatorname{cosec}(-x)\cos(270^\circ-x)\tan(180^\circ+x)}=\cos x$

(m) $\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}=\cot^2 x$ (n) $\frac{\cos x+\sin x}{\cos x-\sin x}-\frac{\cos x-\sin x}{\cos x+\sin x}=2\tan 2x$

(o) $\frac{\cos 4x\sin 3x-\cos 2x\sin x}{\sin 4x\sin x+\cos 6x\cos x}=\tan 2x$ (p) $\frac{\cos 4x+\cos 3x+\cos 2x}{\sin 4x+\sin 3x+\sin 2x}=\cot 3x$

(q) $\frac{\sin 7x+\sin 3x+\sin 9x+\sin 5x}{\cos 7x+\cos 3x+\cos 9x+\cos 5x}=\tan 6x$ (r) $\frac{\sin 5x-2\sin 3x+\sin x}{\cos 5x-\cos x}=\tan x$

(s) $\frac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}=2\sin x$ (t) $\cot 4x(\sin 5x+\sin 3x)=\cot x(\sin 5x-\sin 3x)$

(u) $\cos 4x=1-8\sin^2 x\cos^2 x$ (v) $\cos 6x=32\cos^6 x-48\cos^4 x+18\cos^2 x-1$

(w) $\tan 4x=\frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$ (x) $\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\left(\frac{1+\tan x}{1-\tan x}\right)^2$

(y) $\frac{\sin x-\sin y}{\cos x+\cos y}=\tan\left(\frac{x-y}{2}\right)$ (z) $\frac{\sin(x+y)}{\sin(x-y)}=\frac{\tan x+\tan y}{\tan x-\tan y}$

(aa) $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$ (ab) $\cos 2x\cos\frac{x}{2}-\cos 3x\cos\frac{9x}{2}=\sin 5x\sin\frac{5x}{2}$

(ac) $3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]-2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right]=1$

## Level - III

(a) $\cos 20^\circ\cos 40^\circ\cos 60^\circ\cos 80^\circ=\frac{1}{16}$ (b) $\sin 10^\circ\sin 50^\circ\sin 60^\circ\sin 70^\circ=\frac{\sqrt{3}}{16}$

(c) $\sin 10^\circ\sin 30^\circ\sin 50^\circ\sin 70^\circ=\frac{1}{16}$ (d) $\cos 10^\circ\cos 30^\circ\cos 50^\circ\cos 70^\circ=\frac{3}{16}$

(e) $\cos A\cos 2A\cos 4A\cos 8A=\frac{\sin 16A}{16\sin A}$ (f) $\sin 20^\circ\sin 40^\circ\sin 60^\circ\sin 80^\circ=\frac{3}{16}$

(g) $\cos 20^\circ \cos 40^\circ \cos 80^\circ = \frac{1}{8}$ (h) $\tan 20^\circ \tan 40^\circ \tan 60^\circ \tan 80^\circ = 3$

**Level - IV**

(a) $\cos^3 x + \cos^3(120^\circ + x) + \cos^3(240^\circ + x) = \frac{3}{4}\cos 3x$

(b) $\sin^3 x + \sin^3\left(\frac{2\pi}{3} + x\right) + \sin^3\left(\frac{4\pi}{3} + x\right) = -\frac{3}{4}\sin 3x$

(c) $\cos^2 x + \cos^2\left(x + \frac{\pi}{3}\right) + \cos^2\left(x - \frac{\pi}{3}\right) = \frac{3}{2}$ **OR** $\cos^2 x + \cos^2(x + 60^\circ) + \cos^2(x - 60^\circ) = \frac{3}{2}$

(d) $\cos^2 x + \cos^2\left(x + \frac{2\pi}{3}\right) + \cos^2\left(x - \frac{2\pi}{3}\right) = \frac{3}{2}$

(e) $\sin^2 x + \sin^2\left(x + \frac{2\pi}{3}\right) + \sin^2\left(x - \frac{2\pi}{3}\right) = \frac{3}{2}$ (f) $\sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \sin^2\left(x - \frac{\pi}{3}\right) = \frac{3}{2}$

(g) $\sin A \sin(60^\circ - A)\sin(60^\circ + A) = \frac{1}{4}\sin 3A$ (h) $\cos A \cos(60^\circ - A)\cos(60^\circ + A) = \frac{1}{4}\cos 3A$

(i) $\tan\alpha \tan(60^\circ - \alpha)\tan(60^\circ + \alpha) = \tan 3\alpha$ (j) $\sin A + \sin(A + 120^\circ) + \sin(A + 240^\circ) = 0$

(k) $\cos x + \cos(120^\circ - x) + \cos(120^\circ + x) = 0$ (l) $\tan(60^\circ + \alpha)\tan(60^\circ - \alpha) = \frac{2\cos 2\alpha + 1}{2\cos 2\alpha - 1}$

**Level - V**

(a) $\sin 5A = 5\sin A - 20\sin^3 A + 16\sin^5 A$

(b) $\cos 5x = 16\cos^5 x - 20\cos^3 x + 5\cos x$

(c) $\tan 3x \tan 2x \tan x = \tan 3x - \tan 2x - \tan x$

(d) $\tan 5x \tan 3x \tan 2x = \tan 5x - \tan 3x - \tan 2x$

(e) $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

(f) $(\cos x + \cos y)^2 + (\sin x + \sin y)^2 = 4\cos^2\left(\frac{x - y}{2}\right)$

(g) $(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2\left(\frac{x - y}{2}\right)$

(h) $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4\cos^2 \frac{x + y}{2}$

**Q02.** (a) If $A + B = 45^\circ$, show that $(1 + \tan A)(1 + \tan B) = 2$.

(b) If $A + B = 135^\circ$, show that $(1 - \tan A)(1 - \tan B) = 2$.

**Q03.** If $\cos x + \cos y = \frac{1}{3}$ and $\sin x + \sin y = \frac{1}{4}$, then prove that $\tan\left(\frac{x + y}{2}\right) = \frac{3}{4}$.

**Q04.** If x and y are acute angles such that $\sin x = \frac{1}{\sqrt{5}}$ and $\sin y = \frac{1}{\sqrt{10}}$, prove that $x + y = \frac{\pi}{4}$.

**Q05.** If $\sin x \cos y = \frac{1}{4}$ and $3\tan x = 4\tan y$, then prove that $\sin(x + y) = \frac{7}{16}$.

**Q06.** If $\cos(A + B)\sin(C - D) - \cos(A - B)\sin(C + D) = 0$, then evaluate the following :

$\tan A \tan B \tan C + \tan D$.

**OR** If $\frac{\cos(A + B)}{\cos(A - B)} = \frac{\sin(C + D)}{\sin(C - D)}$, prove that $\tan A \tan B \tan C + \tan D = 0$.

**Q07.** If $\cos(A-B)+\cos(B-C)+\cos(C-A)=-\frac{3}{2}$, then prove the following :

$\cos A+\cos B+\cos C=0=\sin A+\sin B+\sin C$.

**Q08.** If $\tan\frac{\theta}{2}=\sqrt{\frac{a-b}{a+b}}\tan\frac{\phi}{2}$, then show that $\cos\theta=\frac{a\cos\phi+b}{a+b\cos\phi}$.

**Q09.** Prove that $\tan 36^{\circ}+\tan 9^{\circ}+\tan 36^{\circ}\tan 9^{\circ}=1$.

**Q10.** Prove that $\sin(218^{\circ}-x)\cos(38^{\circ}-y)-\cos(218^{\circ}-x)\sin(38^{\circ}-y)=\sin(x-y)$.

**Q11.** Prove that $\sin A+\sin B+\sin C-\sin(A+B+C)=4\sin\frac{A+B}{2}\sin\frac{B+C}{2}\sin\frac{C+A}{2}$.

**Q12.** Prove that $\sin 10^{\circ}+\sin 20^{\circ}+\sin 40^{\circ}+\sin 50^{\circ}=\sin 70^{\circ}+\sin 80^{\circ}$.

**Q13.** If $\sin\alpha\sin\beta-\cos\alpha\cos\beta+1=0$, prove that $1+\cot\alpha\tan\beta=0$.

**Q14.** If $b\sin\beta=a\sin(2\alpha+\beta)$, then prove that $(b+a)\cot(\alpha+\beta)=(b-a)\cot\alpha$.

**Q15.** If $\frac{\sin(x+y)}{\sin(x-y)}=\frac{a+b}{a-b}$, then prove that $\frac{\tan x}{\tan y}=\frac{a}{b}$.

**Q16.** What is the value of $\cot 70^{\circ}+4\cos 70^{\circ}$ ?

**Q17.** **(a)** If $\alpha$ and $\beta$ are distinct roots of $a\cos\theta+b\sin\theta=c$, find the value of $\sin(\alpha+\beta)$.

**(b)** If $\alpha$ and $\beta$ are two different values of $\theta$ lying between $0$ and $2\pi$ which satisfy the equation $6\cos\theta+8\sin\theta=9$, prove that $\sin(\alpha+\beta)=\frac{24}{25}$.

## ✪ Trigonometric Equations & their Solutions*

### 01. Trigonometric equations, General solutions and Principal solutions

An equation involving one or more trigonometric ratios of unknown angle is called a trigonometric equation.

It is important to note that a **trigonometric identity is satisfied for every value of the unknown angle where as trigonometric equation is satisfied for some values (finite or infinite) of unknown angle**.

Since trigonometric functions are periodic functions, therefore, solutions of trigonometric equations can be generalized with the help of **periodicity of trigonometric functions**. The solution consisting of all possible solutions of a trigonometric equation is called its **general solution**.

☑ **General solution of trigonometric equation of following types**

(a) $\sin x=0$ gives $x=n\pi$, where $n\in Z$

(b) $\cos x=0$ gives $x=(2n\pm1)\frac{\pi}{2}$, where $n\in Z$

(c) $\tan x=0$ gives $x=n\pi$, where $n\in Z$

(d) $\sin x=\sin y$ gives $x=n\pi+(-1)^{n}y$, where $n\in Z$

(e) $\cos x=\cos y$ gives $x=2n\pi\pm y$, where $n\in Z$

(f) $\tan x=\tan y$ gives $x=n\pi+y$, where $n\in Z$.

*Look out for the proof of these relations to be discussed in the class!*

**Principal solution :** The solution of a trigonometric equation in x for which $0\le x<2\pi$ that is, $x\in[0, 2\pi)$ are called the principal solutions.

☑ *Did you know?*

(i) $\sin\theta=(-1)^{n}$ *if* $\theta=(2n+1)\frac{\pi}{2}$, $n\in Z$ (ii) $\cos\theta=(-1)^{n}$ *if* $\theta=n\pi$, $n\in Z$

(iii) $\sin\theta = 0$ *if* $\theta = n\pi,\ n \in Z$ (iv) $\cos\theta = 0$ *if* $\theta = (2n+1)\dfrac{\pi}{2},\ n \in Z$.

# WORKED OUT ILLUSTRATIVE EXAMPLES

**Ex01.** Find the principal and general solutions of $\tan x = \dfrac{1}{\sqrt{3}}$.

**Sol.** **Principal solution:** The given equation is $\tan x = \dfrac{1}{\sqrt{3}}$.

We know that, $\tan\dfrac{\pi}{6} = \dfrac{1}{\sqrt{3}}$ and $\tan\left(\pi + \dfrac{\pi}{6}\right) = \dfrac{1}{\sqrt{3}}$

*i.e.*, $\tan\dfrac{\pi}{6} = \dfrac{1}{\sqrt{3}}$ and $\tan\dfrac{7\pi}{6} = \dfrac{1}{\sqrt{3}}$.

Hence, the principal solutions are $x = \dfrac{\pi}{6}$ and $x = \dfrac{7\pi}{6}$.

**General solution:** Given $\tan x = \dfrac{1}{\sqrt{3}}$.

The least value of x in $[0, 2\pi]$ for which $\tan x = \dfrac{1}{\sqrt{3}}$ is $\dfrac{\pi}{6}$.

$\therefore \quad \tan x = \tan\dfrac{\pi}{6}$

$\Rightarrow \quad x = n\pi + \dfrac{\pi}{6}$, where $n \in Z$

Hence, the general solution of $\tan x = \dfrac{1}{\sqrt{3}}$ is $x = n\pi + \dfrac{\pi}{6}$, where $n \in Z$.

**Ex02.** Solve : $\cot^2 x + \dfrac{3}{\sin x} + 3 = 0$.

**Sol.** As $\cot^2 x + \dfrac{3}{\sin x} + 3 = 0$

$\Rightarrow \operatorname{cosec}^2 x - 1 + 3\operatorname{cosec} x + 3 = 0$

$\Rightarrow \operatorname{cosec}^2 x + 3\operatorname{cosec} x + 2 = 0$

$\Rightarrow (\operatorname{cosec} x + 2)(\operatorname{cosec} x + 1) = 0$

$\therefore$ Either $\operatorname{cosec} x + 2 = 0$ or, $\operatorname{cosec} x + 1 = 0$

$\Rightarrow \operatorname{cosec} x = -2$ or, $\operatorname{cosec} x = -1$

$\Rightarrow \sin x = -\dfrac{1}{2}$ or, $\sin x = -1$

$\Rightarrow \sin x = \sin\left(-\dfrac{\pi}{6}\right)$ or, $\sin x = \sin\left(-\dfrac{\pi}{2}\right)$

$\Rightarrow x = n\pi + (-1)^n\left(-\dfrac{\pi}{6}\right)$ or, $x = n\pi + (-1)^n\left(-\dfrac{\pi}{2}\right)\ \forall\ n \in Z$

$\therefore x = n\pi + (-1)^{n+1}\dfrac{\pi}{6},\ n\pi + (-1)^{n+1}\dfrac{\pi}{2}$, where $n \in Z$.

**Ex03.** Solve : $\tan\left(\dfrac{\pi}{4} + \theta\right) + \tan\left(\dfrac{\pi}{4} - \theta\right) = 4$.

**Sol.** We have $\tan\left(\frac{\pi}{4}+\theta\right)+\tan\left(\frac{\pi}{4}-\theta\right)=4$

$$\Rightarrow \frac{\tan\frac{\pi}{4}+\tan\theta}{1-\tan\frac{\pi}{4}\tan\theta}+\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}=4$$

$$\Rightarrow \frac{(1+\tan\theta)^2+(1-\tan\theta)^2}{1^2-\tan^2\theta}=4$$

$$\Rightarrow 2+2\tan^2\theta=4-4\tan^2\theta$$

$$\Rightarrow 6\tan^2\theta=2$$

$$\Rightarrow \tan\theta=\pm\frac{1}{\sqrt{3}}$$

When $\tan\theta=\frac{1}{\sqrt{3}}=\tan\frac{\pi}{6}$ $\Rightarrow \theta=n\pi+\frac{\pi}{6}$ , $n\in Z$

Also, when $\tan\theta=-\frac{1}{\sqrt{3}}=\tan\left(-\frac{\pi}{6}\right)$ $\Rightarrow \theta=n\pi-\frac{\pi}{6}$, $n\in Z$.

Hence, the required solution is $\theta=n\pi\pm\frac{\pi}{6}$, $n\in Z$.

## Exercise 3.4
### Based on Trigonometric Equations

**Q01**. Find the principal and general solution of the following equations:

(a) $\sin x=-\frac{\sqrt{3}}{2}$ (b) $\operatorname{cosec} x=-2$ (c) $\tan x=\sqrt{3}$.

**Q02**. Find the general solution for each of the following:

(a) $\cos x-\sin x=-1$ (b) $\sec x-\tan x=\sqrt{3}$

(c) $\tan x+2=\cot(90^\circ+x)$ (d) $\cos 3x+\cos x-\cos 2x=0$

(e) $\sin x+\sin 3x+\sin 5x=0$ (f) $\sec^2 2x=1-\tan 2x$

(g) $\sin 2x+\cos x=0$ (h) $2\cos^2 x+3\sin x=0$

(i) $\tan 2x=-\cot\left(x+\frac{\pi}{3}\right)$ (j) $\sqrt{3}\cos x-\sin x=1$

(k) $3\tan x+\cot x=5\operatorname{cosec} x$ (l) $\tan x+\tan 2x+\tan 3x=0$

(m) $\cos\theta\cos 2\theta\cos 3\theta=\frac{1}{4}$ (n) $\tan\theta\tan 4\theta\tan 7\theta=\tan\theta+\tan 4\theta+\tan 7\theta$

**Q03**. Solve: $2\sin^2 x+\sqrt{3}\cos x+1=0$.

**Q04**. Solve the equation: $\tan 5\theta=\cot 2\theta$.

**Q05**. Solve the followings:

(a) $4\sin x\cos x+2\sin x+2\cos x+1=0$ (b) $\tan^3 x-3\tan x=0$

(c) $3\tan^3 x-\tan x=0$ (d) $\tan x=\sin x$

(e) $\tan\theta+\tan 2\theta+\sqrt{3}\tan\theta\tan 2\theta=\sqrt{3}$ (f) $\cos 3A+8\cos^3 A=0$.

(g) $(\sqrt{3}+1)\cos\theta+(\sqrt{3}-1)\sin\theta=\sqrt{2}$ (h) $2\sin x+\sqrt{3}\cos x=1+\sin x$

# ☑ Application based and Miscellaneous type problems

**☆ Maximum and Minimum values of $a\sin\theta \pm b\cos\theta$ :**

Let $y = a\sin\theta \pm b\cos\theta$

Put $a = r\cos\alpha,\ b = r\sin\alpha$ s.t. $r^2 = a^2 + b^2$ and $\tan\alpha = \frac{b}{a}$

Now $y = a\sin\theta \pm b\cos\theta = r(\sin\theta\cos\alpha \pm \cos\theta\sin\alpha) = r\sin(\theta \pm \alpha)$

As $-1 \le \sin(\theta \pm \alpha) \le 1$

$\Rightarrow -r \le r\sin(\theta \pm \alpha) \le r$

$\Rightarrow -r \le y \le r$ for all $\theta,\ \alpha \in R$

$\therefore$ Minimum and maximum value of $y = a\sin\theta \pm b\cos\theta$ are respectively $-\sqrt{a^2+b^2}$ and $\sqrt{a^2+b^2}$.

Moreover the Range of y is $y \in \left[-\sqrt{a^2+b^2},\ \sqrt{a^2+b^2}\right]$.

**☑ Keep on tips :**

- $\cos 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4}$
- $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$
- $\sin 36^\circ = \frac{\sqrt{10-2\sqrt{5}}}{4}$
- $\cos 36^\circ = \frac{\sqrt{5}+1}{4}$

## Exercise 3.5

### Application based Questions

**Q01.** If $(1+\tan A)(1+\tan 4A) = 2,\ A \in \left(0, \frac{\pi}{16}\right)$. Find A.

**Q02.** **(a)** If $A = \sin^2\theta + \cos^4\theta$, then what is the range of A?

**(b)** If $y = \cos^2 x + \sin^4 x,\ x \in R$, then prove that $\frac{3}{4} \le y \le 1$.

**Q03.** Find the domain and range of $f(x) = \frac{1}{5 - 2\sin 3x}$.

**Q04.** Draw the graph of $f(x) = 3\cos 2x$. Also write its domain and range.

**Q05.** What is the sign of $\cos\frac{x}{2} - \sin\frac{x}{2}$ when (a) $0 < x < \frac{\pi}{2}$ (b) $\frac{\pi}{2} < x < \pi$.

**Q06.** What is the maximum and minimum value of $3 - 7\cos 5x$?

**Q07.** Evaluate : $\cos 10^\circ + \cos 110^\circ + \cos 130^\circ$.

**Q08.** Simplify : $\frac{\sin(x+y) - 2\sin x + \sin(x-y)}{\cos(x+y) - 2\cos x + \cos(x-y)}$.

**Q09.** Find the value of $\sqrt{3}\,\text{cosec}\,20^\circ - \sec 20^\circ$.

**Q10.** **(a)** If $\tan x + \cot x = 2$, then prove that $\tan^n x + \cot^n x = 2\ \ \forall\ n \in N$.

**(b)** Find all possible values of $\theta$, where $\theta \in (0, \pi)$ such that $\tan^2\theta + \cot^2\theta = 2$.

## Exercise 3.6

### Miscellaneous type Questions

**Q01.** If $\sin\theta = 3\sin(\theta + 2\alpha)$, then find the value of $\tan(\theta+\alpha) + 2\tan\alpha$.

**Q02.** If $x = y\cos\frac{2\pi}{3} = z\cos\frac{4\pi}{3}$, find the value of $xy + yz + zx$.

**Q03.** If $\tan(A - B) = 1$, $\sec(A + B) = \frac{2}{\sqrt{3}}$, then find the smallest value of B, $B > 0$.

**Q04.** Prove that $\tan x + 2\tan 2x + 4\tan 4x + 8\cot 8x = \cot x$.

**Q05.** If $\tan 8^\circ = m$ and $\tan 9^\circ = n$, find the value of $\tan 2^\circ$.

**Q06.** If $0 < \alpha < \frac{\pi}{2}$ and, $\sin\alpha + \cos\alpha + \tan\alpha + \cot\alpha + \sec\alpha + \text{cosec}\,\alpha = 7$, then show that $\sin 2\alpha$ is a root of the equation $x^2 - 44x + 36 = 0$.

**Q07.** If $\sin(x + y) = \frac{p}{\sqrt{1 + p^2}}$ and $\cos(x - y) = \frac{1}{\sqrt{1 + q^2}}$, then show that $\tan x$ is a root of the equation $(p + q)z^2 + 2(1 - pq)z - (p + q) = 0$.

***Q08.** Solve : $\cos x \cos 2x + 1 = 0$.

**Q09.** If $\sin\alpha = \sin\beta$ and $\cos\alpha = \cos\beta$, find the value of $\sin\left(\frac{\alpha - \beta}{2}\right)$.

***Q10.** Solve : $\sec\theta \cos 5\theta + 1 = 0$.

**Q11.** Write the value of $\cos\frac{\pi}{5}\cos\frac{2\pi}{5}\cos\frac{4\pi}{5}\cos\frac{8\pi}{5}$.

**Q12.** Draw sin x, sin 2x, and sin 3x on the same graph and with same scale.

**Q13.** Evaluate : $\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8}$.

**Q14.** If $\cos x = \cos\alpha\cos\beta$ then show that $\tan\left(\frac{x + \alpha}{2}\right)\tan\left(\frac{x - \alpha}{2}\right) = \tan^2\frac{\beta}{2}$.

**Q15.** If $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$ then, show that $\cos\left(\theta - \frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}}$.

**Q16.** Solve : $81^{\sin^2 x} + 81^{\cos^2 x} = 30,\ 0 < x < \pi$.

**Q17.** If $\tan x + \tan y = a$ and $\cot x + \cot y = b$, then find the value of $\cot(x + y)$.

**Q18.** Find the number of solutions for the equation $\tan x + \sec x = 2\cos x,\ x \in [0, 2\pi]$.

**Q19.** What is the value of $\cos^2 48^\circ - \sin^2 12^\circ$?

**Q20.** The number of sides of two regular polygons are 5 : 4 and the difference between their angles is $9^\circ$. Find the number of sides of the polygons.

**Q21.** Perimeter of a certain sector of a circle is equal to half that of the circle of which it is a sector. Find the circular measure of the angle of the sector.

**Q22.** Find the angle between the minute and hour hands of a clock at 3 : 40.

**Q23.** (a) Show that $\sin^2\theta = \frac{(a + b)^2}{4ab}$ is possible only when $a = b$.

(b) Find the values of $\cos\theta$ for which $2\cos\theta = a + \frac{1}{a}$ is possible, where $a \in R$

(c) Prove that $\sec^2 A + \text{cosec}^2 A \geq 4$.

(d) Is the equation $2\sin^2\theta - \cos\theta + 4 = 0$ possible?

(e) Find the value of $2\cos 67\frac{1}{2}^\circ \cos 22\frac{1}{2}^\circ$.

(f) If $m\tan\theta = n$, then prove that $m\cos 2\theta + n\sin 2\theta = m$.

(g) Find the value of $x \in \left(0, \dfrac{\pi}{2}\right)$, if $2^{\cos^2 x} + 2^{\sin^2 x} = 2\sqrt{2}$.

(h) If $2\cos^2 x + \cos^2 2x = 2$, find cos x, where x lies in I quadrant.

**Q24.** Reduce $\sqrt{3}\sin x - \cos x$ as a single term consisting (i) sine only (ii) cosine only.

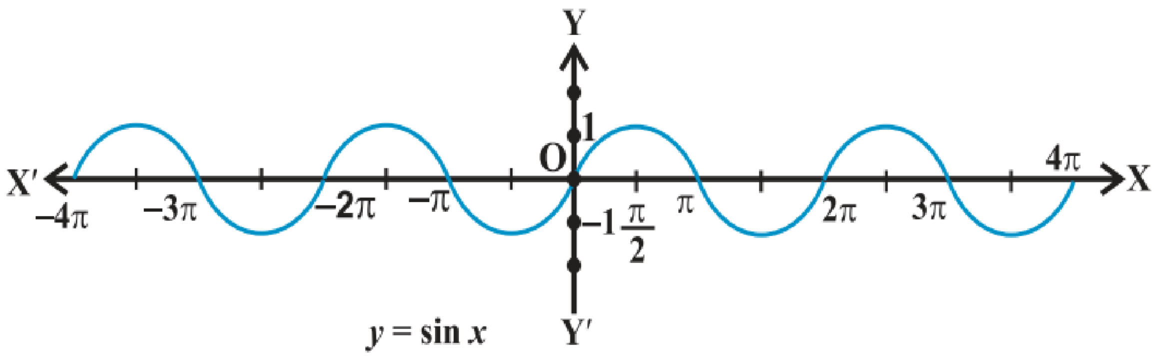
**Q25.** Find a pair of values of R and x from $R\sin x = 1$ and $R\cos x = \sqrt{3}$.

***Q26.** Find the angle x, if $3\tan(x - 15^\circ) = \tan(x + 15^\circ)$.

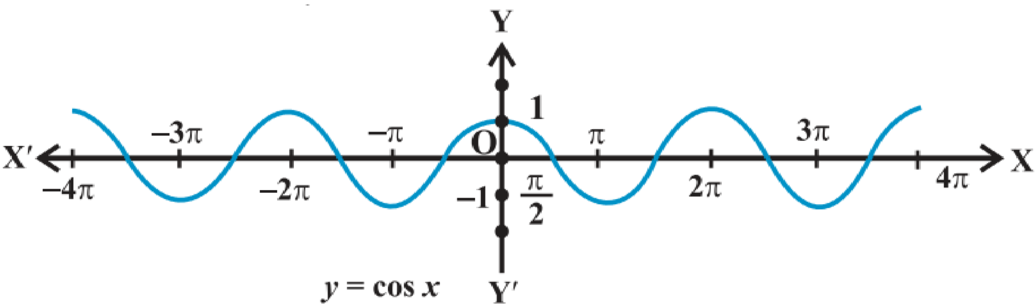
## ✪ Graphs of Trigonometric Functions

Here we're presenting the Graphs of standard trigonometric functions, for your reference.

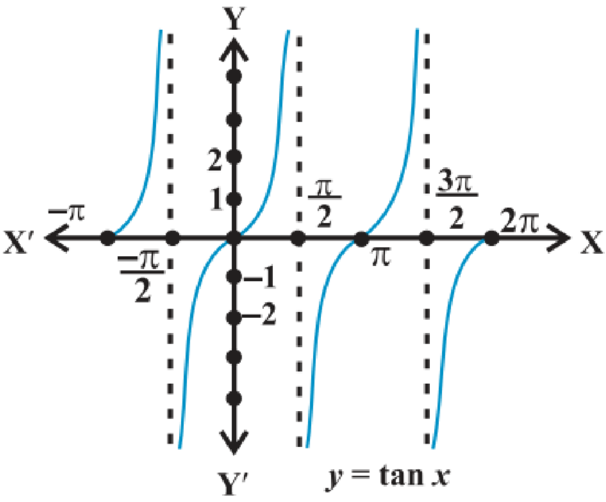
☑ Graph of $y = \sin x$



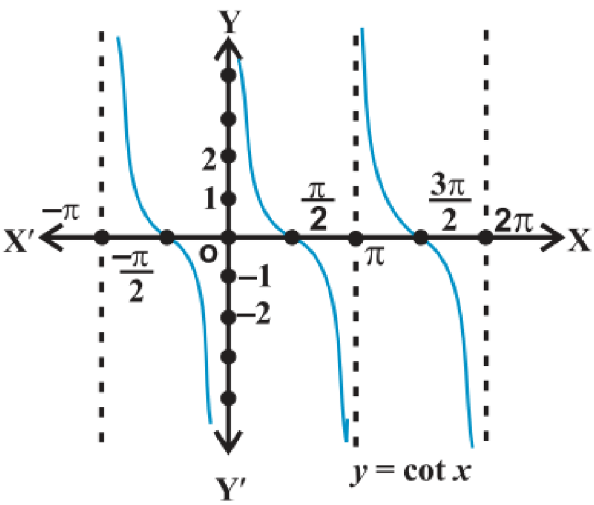

☑ Graph of $y = \cos x$



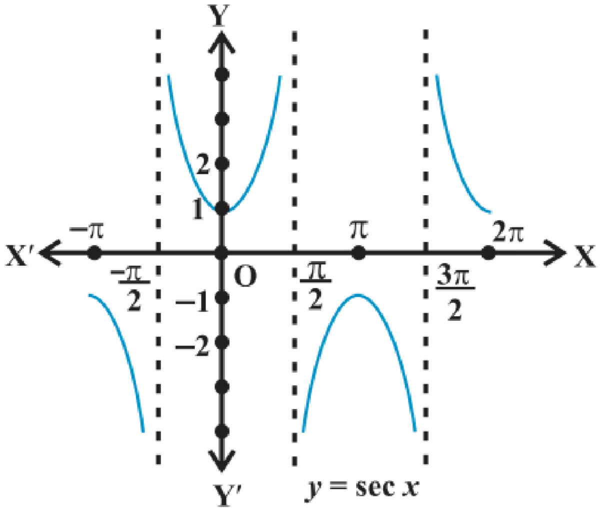

☑ Graph of $y = \tan x$



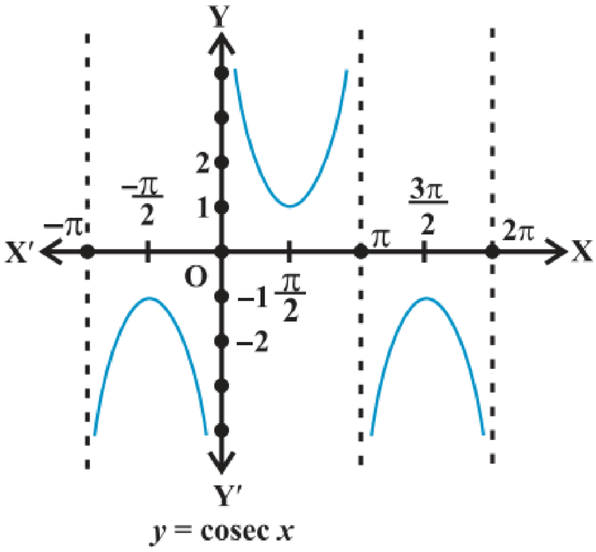

☑ Graph of $y = \cot x$



☑ Graph of $y = \sec x$




☑ Graph of $y = \text{cosec}\, x$




*Curve sketching is* ***very important*** *in understanding the behavior of functions.*
*Therefore, it is advised to learn and practice the plotting of graphs.*
*Teachers may explain the graph of t-functions for the benefit of students.*

# MATH WARRIORS TEST-03 ✍

**Time Allowed : 60 Minutes** **Max. Marks : 35**

Q01. Evaluate $\tan\frac{13\pi}{12}$.

Q02. Prove that $\sin 51^{\circ}+\cos 81^{\circ}=\cos 21^{\circ}$.

Q03. Determine the value of $\cot 75^{\circ}+\tan 75^{\circ}$. [1×3

Q04. Find the angle between the minute and hour hands of a clock at 3 : 40.

Q05. Prove that $A-B=60^{\circ}$, if $\cos A=\frac{1}{7}$, $\cos B=\frac{13}{14}$.

Q06. (a) What is the maximum and minimum values of $24\sin x+7\cos x$.

(b) Prove that $\tan 189^{\circ}=\frac{\cos 36^{\circ}-\sin 36^{\circ}}{\cos 36^{\circ}+\sin 36^{\circ}}$.

Q07. Prove that $\frac{\sin(A-B)}{\cos A\cos B}+\frac{\sin(B-C)}{\cos B\cos C}+\frac{\sin(C-A)}{\cos C\cos A}=0$.

Q08. Prove that $\tan 13A-\tan 9A-\tan 4A=\tan 13A\tan 9A\tan 4A$. [4×5

Q09. If $\tan x=\frac{3}{4}$, $\pi<x<\frac{3\pi}{2}$, find the values of $\text{cosec}\frac{x}{2}$, $\sec\frac{x}{2}$ and $\cot\frac{x}{2}$.

Q10. Find the value of $\cos 10^{\circ}\cos 50^{\circ}\cos 60^{\circ}\cos 70^{\circ}$. [6×2

## ☑ ANSWERS

Q01. $2-\sqrt{3}$

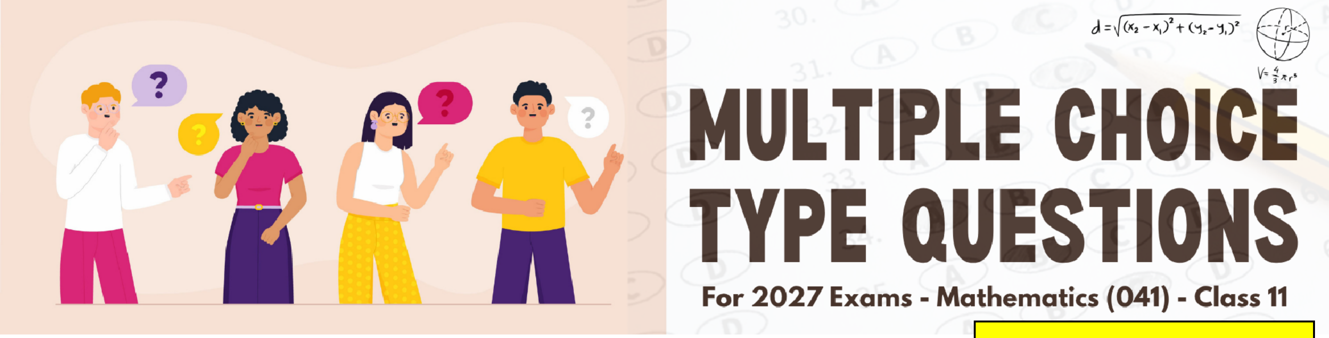
Q03. 4

Q04. $130^{\circ}$

Q06. (a) 25, –25 (b) Obtain $\frac{1-\tan 36^{\circ}}{1+\tan 36^{\circ}}=\tan 9^{\circ}=\tan 189^{\circ}$

Q09. $\frac{\sqrt{10}}{3}$, $-\sqrt{10}$ and $-\frac{1}{3}$

Q10. $\frac{\sqrt{3}}{16}$.

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## ☑ Chapter 01
## Sets Theory

01. $\{x : x$ is a two digit number so that the sum of its digits is one$\}$ in the tabular form, is given by
(a) $\{10\}$, $\{01\}$ both (b) $\{100\}$ (c) $\{10\}$ (d) $\{01\}$

02. If $A = \{0\}$, then A is
(a) null set (b) infinite set (c) singleton set (d) disjoint set

03. For $X = \{2, 4, 6\}$ and $Y = \{1, 3, 6, 10, 15\}$, $X - Y =$
(a) $\{2, 4\}$ (b) $\{2, 4, 6\}$ (c) $\{1, 3, 10, 15\}$ (d) $\phi$

04. If U is a universal set and A is a non-empty set then, which of the following is not true?
(a) $A \cup U = A$ (b) $A \cup U = U$ (c) $A \cap U = A$ (d) $A \cap A' = \phi$

05. If U is a universal set and A is a non-empty set then, which of the following is true?
(a) $A \cup U = A$ (b) $A \cup A' = A$ (c) $A \cap A' = \phi$ (d) $A \cap U' = A$

## ☑ Chapter 02
## Relations & Functions

01. Domain of $f(x) = \dfrac{1}{\sqrt{x^2 - 5x - 6}}$ is
(a) $\mathbb{R}$ (Real nos.) (b) $\mathbb{R} - [-1, 6]$ (c) $\mathbb{R} - \{-1\}$ (d) $\mathbb{R} - \{-1, 6\}$

02. If $|x| \geq 3$, then $x \in$
(a) $(-3, 3)$ (b) $[-3, 3]$ (c) $(-\infty, -3) \cup (3, \infty)$ (d) $(-\infty, -3] \cup [3, \infty)$

03. If $A = \{1, 2, 3, 4\}$ and $B = \{5, 6, 7\}$, then no. of functions defined from A to B is
(a) 64 (b) 81 (c) 4096 (d) 144

04. For the function $f(x) = [x]$, where [.] is greatest integer function, the range of $f(x)$ is
(a) $Z^+$ (b) $Z^-$ (c) $[0, \infty)$ (d) Z

05. If $A = \{1, 2, 3\}$, $B = \{4, 5\}$ then, a relation R defined from A to B, having maximum no. of elements is given by
(a) $B \times B$ (b) $A \times A$ (c) $A \times B$ (d) $B \times A$

## ☑ Chapter 03
## Trigonometric Functions

01. The greatest value of $\sin x \cos x$ is
(a) 1 (b) 2 (c) $\sqrt{2}$ (d) $\dfrac{1}{2}$

02. The value of $\tan 0° \times \tan 1° \times \tan 2° \times \tan 3° \times ... \times \tan 89°$ is
(a) 0 (b) 1 (c) $\dfrac{1}{2}$ (d) Not defined

03. The value of $\cos 1° \times \cos 2° \times \cos 3° \times ... \times \cos 179°$ is

(a) $\frac{1}{\sqrt{2}}$ (b) 1 (c) 0 (d) −1

04. The value of $\frac{1-\tan^2 15°}{1+\tan^2 15°}$ is

(a) 1 (b) $\sqrt{3}$ (c) $\frac{\sqrt{3}}{2}$ (d) 2

05. The value of $\sin 50° - \sin 70° + \sin 10°$ is equal to

(a) 1 (b) 0 (c) $\frac{1}{2}$ (d) 2

06. If $\sin\theta + \cos\theta = 1$, then the value of $\sin 2\theta$ is equal to

(a) 1 (b) 1 (c) 0 (d) 2

## ☑ Chapter 04
## Complex Numbers

01. $(\sqrt{-2})(\sqrt{3})$ is equal to

(a) $\sqrt{6}$ (b) $-\sqrt{6}$ (c) $i\sqrt{6}$ (d) $i2\sqrt{3}$

02. If $\frac{(a^2+1)^2}{2a-i} = x+iy$, then $x^2+y^2 =$

(a) $\frac{(a^2+1)^4}{4a^2+1}$ (b) $\frac{(a+1)^2}{4a^2+1}$ (c) $\frac{(a^2+1)^2}{(4a^2-1)^2}$ (d) None of these

03. If $z = \frac{1}{1-\cos\theta - i\sin\theta}$, then $\text{Re}(z) =$

(a) 0 (b) $\frac{1}{2}$ (c) $\cot\frac{\theta}{2}$ (d) $\frac{1}{2}\cot\frac{\theta}{2}$

04. If $f(z) = \frac{7-z}{1-z^2}$, where $z = 1+2i$, then $|f(z)|$ is

(a) $\frac{|z|}{2}$ (b) $|z|$ (c) $2|z|$ (d) None of these

## ☑ Chapter 05
## Linear Inequations

01. For the inequation $\frac{3(x-2)}{5} \geq \frac{5(2-x)}{3}$, $x \in$

(a) $(\infty, 2]$ (b) $[2, \infty)$ (c) $(-\infty, 2]$ (d) $(\infty, 2)$

02. Consider $4x+3 < 5x+7$. Then $x \in$

(a) $(4, \infty)$ (b) $(-4, \infty)$ (c) $(2, \infty)$ (d) $(-2, \infty)$

03. For $3x - 2 < \frac{x}{3}$, we always have $x \in$

(a) $\left(\frac{3}{4}, \infty\right)$ (b) $\left(-\frac{3}{4}, \infty\right)$ (c) $\left(-\infty, \frac{3}{4}\right)$ (d) $\left(-\infty, \frac{3}{4}\right]$

04. **Fill in the blanks:** If $a < b$ and $c < 0$, then $\left(\frac{a}{c}\right)$ ______ $\left(\frac{b}{c}\right)$.

(a) < (b) ≤ (c) > (d) ≥

## ☑ Chapter 06
## Permutations & Combinations

01. What is the number of ways of arrangement of letters of word BANANA so that no two N's are together?
(a) 40 (b) 60 (c) 80 (d) 100

02. What is the value of n, if ${}^{15}P_{n-1} : {}^{16}P_{n-2} = 3:4$?
(a) 10 (b) 12 (c) 14 (d) 15

03. If 7 points out of 12 are in the same straight line, then what is the number of triangles formed?
(a) 84 (b) 175 (c) 185 (d) 201

04. In how many ways can a bowler take four wickets in a single 6 balls over?
(a) 6 (b) 15 (c) 20 (d) 30

## ☑ Chapter 07
## Binomial Theorem

01. The middle term in the expansion of $\left[2x - \frac{1}{3x}\right]^{10}$; $x \neq 0$ is
(a) ${}^{10}C_4 \frac{2^4}{3^4}$ (b) $-{}^{10}C_5 \frac{2^5}{3^5}$ (c) $-{}^{10}C_4 \frac{2^4}{3^5}$ (d) ${}^{10}C_5 \frac{2^5}{3^5}$

02. For all $n \in N$, $2^{4n} - 15n - 1$ is divisible by
(a) 125 (b) 225 (c) 450 (d) 625

03. What is the coefficient of $x^n$ in the expansion of $(x^2 + 2x)^{n-1}$?
(a) $(n-1) \times 2^{(n-2)}$ (b) $(n-1) \times 2^{(n-1)}$ (c) $(n-1) \times 2^n$ (d) $n \times 2^{(n-1)}$

04. The coefficient of $x^{-3}$ in the expansion of $\left[x - \frac{m}{x}\right]^{11}$; $x \neq 0$ is
(a) $-924m^7$ (b) $-792m^5$ (c) $-792m^6$ (d) $-330m^7$

## ☑ Chapter 08
## Sequences & Series

01. Let A and G be the arithmetic mean and geometric mean of two positive nos., then which of the following is true?
(a) $G \geq A$ (b) $A = \sqrt{G}$ (c) $A \geq G$ (d) $G = \sqrt{A}$

02. The third term of G.P. is 4. The product of its first 5 terms is
(a) $4^3$ (b) $2^8$ (c) $2^{10}$ (d) $\frac{1}{4^5}$

03. If x, 2y, 3z are in A.P., where the distinct numbers x, y, z are in G.P., then the common ratio of the G.P. is
(a) 3 (b) $\frac{1}{3}$ (c) 2 (d) $\frac{1}{2}$

04. The minimum value of $4^x + 4^{1-x}$, $x \in R$ is
(a) 2 (b) 4 (c) 1 (d) 0

## ☑ Chapter 09
## Straight Lines

01. The angle between the straight lines $x - y\sqrt{3} = 5$ and $\sqrt{3}x + y = 7$ is
(a) 90° (b) 60° (c) 75° (d) 30°

02. If p is the length of the perpendicular drawn from the origin to the line $\frac{x}{a}+\frac{y}{b}=1$, then which one of the following is correct?

(a) $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$ (b) $\frac{1}{p^2}=\frac{1}{a^2}-\frac{1}{b^2}$ (c) $\frac{1}{p}=\frac{1}{a}+\frac{1}{b}$ (d) $\frac{1}{p}=\frac{1}{a}-\frac{1}{b}$

03. What is the equation of the line passing through $(2,-3)$ and parallel to y-axis?

(a) $y=-3$ (b) $y=2$ (c) $x=2$ (d) $x=-3$

04. If the lines $3x+4y+1=0$, $5x+\lambda y+3=0$ and $2x+y-1=0$ are concurrent, then $\lambda$ is equal to

(a) $-8$ (b) 8 (c) 4 (d) $-4$

05. The x-intercept and the y-intercept of the line $5x-7=6y$, respectively are

(a) 5 and 6 (b) $\frac{7}{5}$ and $-\frac{7}{6}$ (c) $\frac{5}{7}$ and $\frac{6}{7}$ (d) $-\frac{5}{7}$ and $\frac{6}{7}$

## ☑ Chapter 10
## Conic Sections

01. The equation of the circle which passes through the points of intersection of the circles $x^2+y^2-6x=0$ and $x^2+y^2-6y=0$; and has its centre at $\left(\frac{3}{2},\frac{3}{2}\right)$ is

(a) $x^2+y^2+3x+3y+9=0$ (b) $x^2+y^2+3x+3y=0$

(c) $x^2+y^2-3x-3y=0$ (d) $x^2+y^2-3x-3y+9=0$

02. Value of p, for which $x^2+y^2-2px+4y-12=0$ represents a circle of radius 5 units is

(a) 3 (b) $-3$ (c) both (a) and (b) (d) Neither (a) nor (b)

03. The eccentricity of the ellipse $9x^2+25y^2=225$ is e, then the value of '5e' is

(a) 3 (b) 4 (c) 2 (d) 1

04. The centre of the circle $x^2+y^2-6x+4y-12=0$ is $(a, b)$, then $(2a+3b)$ is

(a) 0 (b) 2 (c) 3 (d) 5

## ☑ Chapter 11
## Introduction to Three Dimensional Geometry

01. A point on zx-plane which is equidistant from the points $(1,-1,0)$, $(2,1,2)$, $(3,2,-1)$ is

(a) $\left(\frac{1}{5},0,\frac{31}{10}\right)$ (b) $\left(\frac{1}{10},0,\frac{31}{5}\right)$ (c) $\left(\frac{31}{10},0,\frac{1}{5}\right)$ (d) $\left(\frac{31}{5},0,\frac{1}{10}\right)$

02. A point on y-axis which is at a distance of $\sqrt{10}$ from the point $(1,2,3)$ is

(a) $(2,0,2)$ (b) $(0,2,2)$ (c) $(2,2,2)$ (d) $(0,2,0)$

03. The locus of a point for which $y=0, z=0$ is

(a) x-axis (b) y-axis (c) z-axis (d) y and z axes

04. A line is parallel to xy-plane, if all points on the line have equal

(a) x-coordinates (b) y-coordinates (c) z-coordinates (d) x and y coordinates

## ☑ Chapter 12
## Limits & Derivatives

01. $\lim_{x\to\pi}\left(\frac{\sin x}{x-\pi}\right)=$

(a) 1 (b) 2 (c) $-1$ (d) does not exist

02. If $\lim_{x\to 2}\frac{x^n-2^n}{x-2}=80$, then n is

(a) 2 (b) 3 (c) 4 (d) 5

03. If $L=\lim_{x\to 1}\frac{x^4-1}{x^3-1}$, then value of $3L$ is

(a) 2 (b) 3 (c) 4 (d) 1

04. $\lim_{x\to 0}\frac{(1+x)^{16}-1}{(1+x)^4-1}=$

(a) 0 (b) 4 (c) 8 (d) 16

05. $\lim_{x\to 1}\frac{x+x^2+x^3+x^4-4}{x-1}$ is

(a) 0 (b) 4 (c) 10 (d) does not exist

06. $\lim_{x\to\frac{\pi}{4}}\frac{\sec^2 x-2}{\tan x-1}$ is

(a) 0 (b) 1 (c) 2 (d) 4

## ☑ Chapter 13
## Statistics

01. The variance of 10 observations is 16 and their mean is 12. If each observation is multiplied by 4, what is the new mean?

(a) 12 (b) 16 (c) 24 (d) 48

02. The variance of 10 observations is 16 and their mean is 12. If each observation is multiplied by 4, what is the new standard deviation?

(a) 4 (b) 8 (c) 16 (d) 32

03. The standard deviation of 25 observations is 4 and their mean is 25. If each observation is increased by 10, what is the new mean?

(a) 25 (b) 29 (c) 30 (d) 35

04. The standard deviation of 35 observations is 4 and their mean is 25. If each observation is increased by 10, what is the new variance?

(a) 4 (b) 14 (c) 16 (d) 25

05. Consider the following table.
Given that the mean of $x_1, x_2, ..., x_{20}$ is 10.

| | **COLUMN 1** | | **COLUMN 2** |
|---|---|---|---|
| A | Mean of $2x_1, 2x_2, ..., 2x_{20}$ | P | 0 |
| B | Mean of $(-3x_1+32), (-3x_2+32), ..., (3x_{20}+32)$ | Q | 2 |
| C | Mean of $(x_1+2), (x_2+2), ........., (x_{20}+2)$ | R | 12 |
| D | Mean of $(x_1-10), (x_2-10), ......., (x_{20}-10)$ | S | 20 |

(a) $A\to P, B\to Q, C\to R, D\to S$ (b) $A\to S, B\to Q, C\to R, D\to P$

(c) $A\to Q, B\to S, C\to R, D\to P$ (d) $A\to S, B\to Q, C\to P, D\to R$

## ☑ Chapter 14
## Probability

01. Without repetition of the digits, four digit numbers are formed with the numbers 0, 2, 3, 5.

The probability of such a number divisible by 5 is

(a) $\frac{1}{5}$ (b) $\frac{4}{5}$ (c) $\frac{5}{9}$ (d) $\frac{1}{30}$

02. Three digit numbers are formed using the digits 0, 2, 4, 6, 8. A number is chosen at random out of these numbers. What is the probability that this number has the same digits?

(a) $\frac{1}{12}$ (b) $\frac{1}{16}$ (c) $\frac{4}{65}$ (d) $\frac{1}{25}$

03. The probability that a non-leap year selected at random will have 52 Sundays is

(a) 0 (b) 1 (c) $\frac{1}{7}$ (d) $\frac{2}{7}$

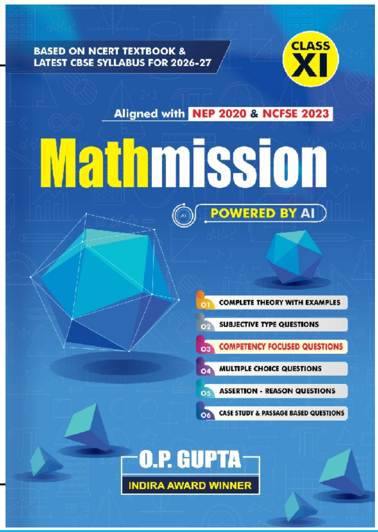
04. The probability that a non-leap year selected at random will have 53 Sundays is

(a) 0 (b) 1 (c) $\frac{1}{7}$ (d) $\frac{2}{7}$

16. If the probabilities for A to fail in an examination is 0.2 and that for B is 0.3, then the probability that either A or B fails is

(a) $>\frac{1}{2}$ (b) $\frac{1}{2}$ (c) $\leq\frac{1}{2}$ (d) 0





# ASSERTION REASON Type Questions

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In the following questions, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct answer out of the following choices.

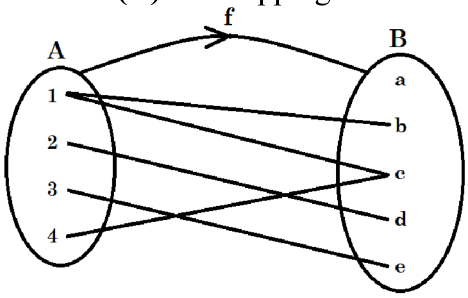
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

## *Unit-I : Sets and Functions*

01. **Assertion (A) :** Set $\{1, 2, 3, 4\}$ and $\{3, 2, 4, 1\}$ are equal sets.
**Reason (R) :** Two sets $A$ and $B$ are said to be equal if they have exactly the same elements.

02. **Assertion (A) :** If $X=\{1,2,3\}$, $Y=\{4,5,6\}$, then $X\cup Y=\{1,2,3,4,5,6\}$.
**Reason (R) :** $P\cap Q=\{x : x\in P \text{ and } x\in Q\}$.

09. **Assertion (A) :** A mapping shown in the following figure is not a function.




**Reason (R) :** A relation $f$ from a set of $A$ to a set $B$ is said to be a function if every element of set $A$ has one and only one image in $B$.

13. **Assertion (A) :** $\sin(-390^\circ)=-\frac{1}{2}$.
**Reason (R) :** $\tan 2x=\frac{2\tan x}{1+\tan^2 x}$.

25. **Assertion (A) :** For $\left[-\pi,-\frac{\pi}{2}\right)$, the length of the interval is $\frac{\pi}{2}$.
**Reason (R) :** The number $(b-a)$ is called the *length* of any of the intervals (a, b), [a, b], [a, b) or (a, b].

36. **Assertion (A) :** $\sin 2025\pi=0$.
**Reason (R) :** $\sin(n\pi)=0,\ n\in Z$.

## *Unit-II : Algebra*

01. **Assertion (A) :** For $z=3-i\sqrt{2}$, $|z|=\sqrt{11}$.
**Reason (R) :** $|z|=\sqrt{x^2+y^2}$, if $z=x+iy$.

04. **Assertion (A) :** For $2(2x+3)-10\le 6(x-2)$, $x\in[4,\infty)$.
**Reason (R) :** For $-5\le\frac{2-3x}{4}\le 9$, $x\in\left[-\frac{34}{3},\frac{22}{3}\right]$.

06. **Assertion (A) :** ${}^{10}C_3=120$.

**Reason (R) :** ${}^{n}C_{r}=\dfrac{n!}{(n-r)!}$.

10. **Assertion (A) :** Third term in $\left(\dfrac{x}{3}+\dfrac{1}{x}\right)^{5}$, $x\neq 0$ is given by $\dfrac{10x}{27}$.

**Reason (R) :** In the binomial $(a+b)^{n}$, $T_{r+1}={}^{n}C_{r}\,b^{r}\,a^{n-r}$.

14. **Assertion (A) :** $2,8,32,...$ forms a geometric progression with common ratio of $\dfrac{1}{4}$.

**Reason (R) :** In a geometric progression $a_1,a_2,a_3,...,a_{n-1},a_n$, we always have $\dfrac{a_2}{a_1}=\dfrac{a_3}{a_2}=...=\dfrac{a_n}{a_{n-1}}=r$, where $r$ is the common ratio of geometric progression.

20. **Assertion (A) :** The infinite geometric progression $a,ar,ar^2,...$ upto $\infty$, sum to infinity exists only if $|r|<1$.

**Reason (R) :** In a geometric progression with first term given as 'a' if common ratio $r>1$, then the sum to n terms of the G.P. is $S_n=\dfrac{a(1-r^n)}{1-r}$.

## *Unit-III : Coordinate Geometry*

01. **Assertion (A) :** Slope of line $y-x-3=0$ is 1.

**Reason (R) :** For the line $y=mx+c$, slope is given by m.

05. **Assertion (A) :** The centre of circle $x^2+y^2-2x+4y=0$ is located at (1, –2).

**Reason (R) :** For the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$, the eccentricity is always greater than 1.

08. **Assertion (A) :** Distance of the point (1, –1, 1) from origin is $\sqrt{3}$ units.

**Reason (R) :** If $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$, then $AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$.

25. **Assertion (A) :** For $\dfrac{x^2}{36}+\dfrac{y^2}{9}=1$, the distance between foci is $6\sqrt{3}$.

**Reason (R) :** The length of minor axis of ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$, $(a>b)$ is given by '2a' units.

26. **Assertion (A) :** The point (1, –1, 1) is at a distance of $\sqrt{3}$ units from (0, 0, 0).

**Reason (R) :** The distance of (x, y, z) from origin is $\sqrt{x^2+y^2+z^2}$.

## *Unit-IV : Calculus*

01. **Assertion (A) :** $\dfrac{d}{dx}\left[x^5-2x^4+5\right]=5x^4-8x^3$.

**Reason (R) :** Differentiation of $x^n$ with respect to x is $nx^{n-1}$.

06. **Assertion (A) :** $\dfrac{d}{dx}\left[\left(\dfrac{x^a}{x^b}\right)^{a+b}.\left(\dfrac{x^b}{x^c}\right)^{b+c}.\left(\dfrac{x^c}{x^a}\right)^{c+a}\right]=0$.

**Reason (R) :** Derivative of a constant function is zero.

07. **Assertion (A) :** $\lim\limits_{x\to 0}\dfrac{\sin 2x}{x}=2$.

**Reason (R) :** $\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1$.

11. **Assertion (A) :** For $f(x) = 2x^2 + 3x - 5$, we get $f'(0) + 3f'(-1) = 0$.

**Reason (R) :** $\left(\frac{u}{v}\right)' = \frac{vu' + uv'}{v^2}$, provided the functions $u$ and $v$ both are defined; $v \neq 0$.

## *Unit-V : Statistics and Probability*

01. **Assertion (A) :** The variance of 5, 5, 5, 5 is zero.

**Reason (R) :** Variance $(\sigma^2) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2$.

03. In the throw of a die, events $A = \{x : x$ is an even number$\}$ and $B = \{x : x$ is an odd number$\}$.

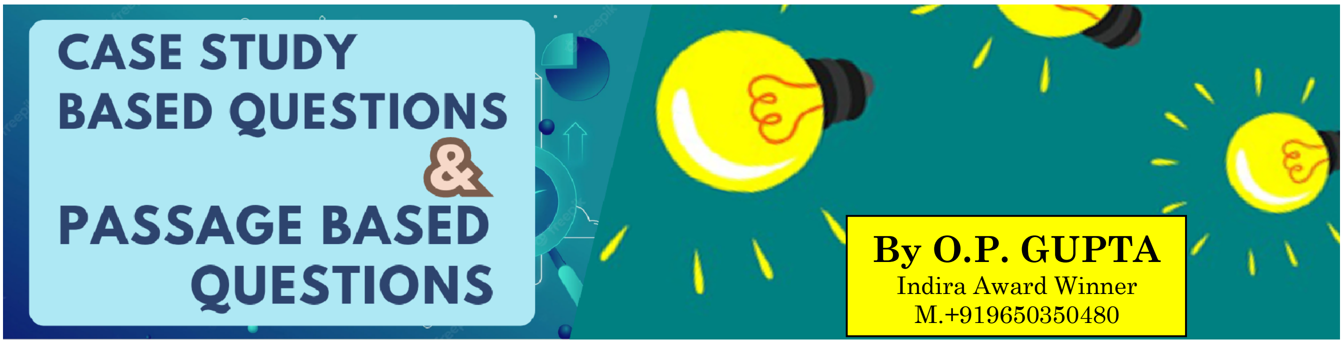
**Assertion (A) :** Event $A = \{2, 4, 6\}$ and event $B = \{1, 3, 5\}$.

**Reason (R) :** Two events A and B are mutually exclusive events if $A \cap B \neq \phi$.

08. Let $P(E) = \frac{3}{7}$, $P(\overline{F}) = \frac{1}{2}$ and $P(\overline{E} \cap \overline{F}) = \frac{1}{14}$.

**Assertion (A) :** The events E and F are mutually exclusive events.

**Reason (R) :** $P(E \cap F) = 0$, with respect to the data mentioned above.




## ☑ Chapter 01
## Sets Theory

Q01. In a city school during the admission to class XI, 18 students took English, 23 students took Hindi and 24 students took Sanskrit.
Of these, 13 took both Hindi and Sanskrit, 12 took both English and Hindi and 11 took both English and Sanskrit.
Due to the request made by some students, the school authorities decided that 6 students will be offered all the three languages.


Based on the above information answer the following questions.

(i) Find the total number of students who took admission in class XI.
(ii) How many students took Sanskrit but not Hindi?
(iii) How many students took exactly one of the three subjects?
(iv) How many students took exactly two of the three subjects?
(v) How many students took Hindi but not Sanskrit?

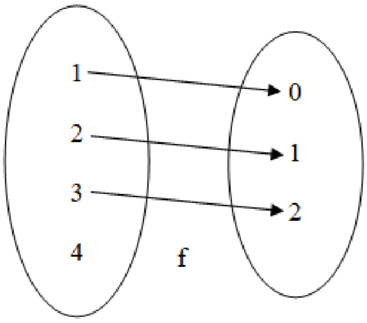
Q02. If every element of a set A is also an element of a set B, then set A is called a subset of B and we write $A \subseteq B$. Thus, $A \subseteq B \Leftrightarrow \{x \in A \Rightarrow x \in B\}$.
Moreover if $n(A) = m$, then set A will have a total of $2^m$ subsets.
Based on the above information, answer the following questions.

(i) How many subsets are possible for set $A = \{-2, -1, 0\}$?
(ii) Let $E = \{\ \}$. Is E a subset of $A = \{-2, -1, 0\}$? Justify your answer.
(iii) Write all those subsets of $A = \{-2, -1, 0\}$, which have exactly two elements.
(iv) Write all those subsets of $A = \{-2, -1, 0\}$, which have exactly one elements.
(v) Write all the subsets of $A = \{-2, -1, 0\}$.

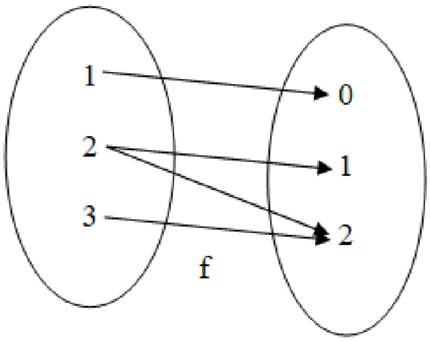
## ☑ Chapter 02
## Relations & Functions

Q01. Given a relation in $x$ and $y$, we say '$y$ is a function of $x$' if for every element $x$ in the domain, there corresponds exactly one element $y$ in the range.
Based on the above information, attempt the following questions.

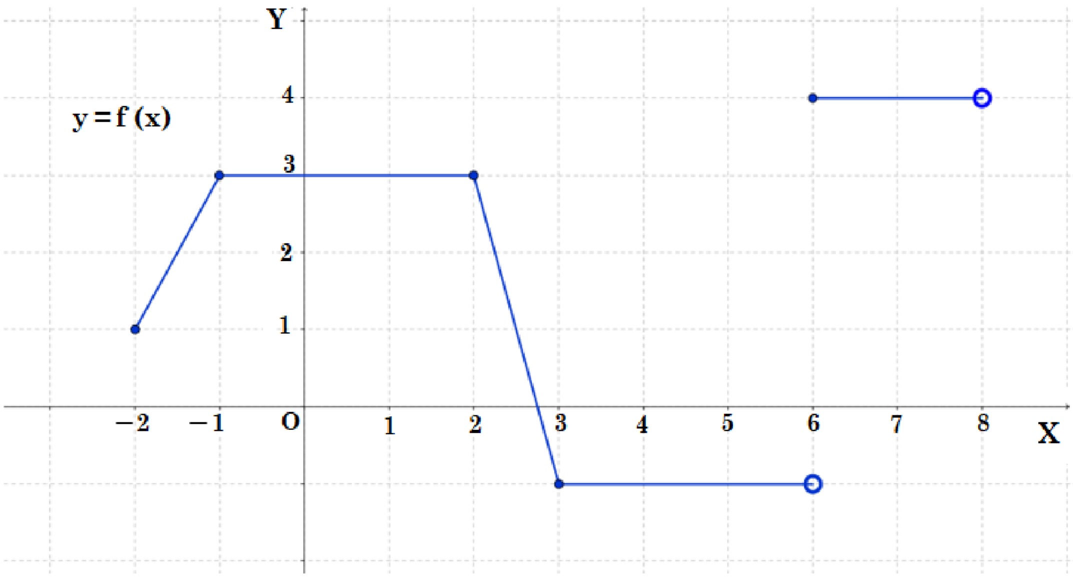
(i) Determine whether the following is a function or not. Justify your answer.




(ii) Determine whether the following is a function or not. Justify your answer.




(iii) Determine the domain and range of the function $y = f(x)$, whose graph is shown below.




(iv) Examine the graph shown in (iii). Mention the integral value(s) of $x$ at which $f(x) = 3$.

(v) Check if $f = \{(a, z), (b, y), (b, x), (c, w), (d, v)\}$ is a function or not. Justify your answer.

Q02. A relation R defined from A to B, is a subset of $A \times B$.
Based on the above information, attempt the following questions.

(i) If $A = \{1, 2, 3\}$, $B = \{4, 5\}$, then write a relation R defined from A to B, having maximum number of elements.

(ii) For the data given in (i), what will be the total number of relations?

(iii) Check if $S : A \to B$, where $S = \{(1,4), (2,5), (3,4), (4,5)\}$ is a relation or not. Give reason.

(iv) For a relation $R' : B \to A$ defined as $R' = \{(x, y) : x \in B,\ y \in A;\ x \text{ is divisible by } y\}$, write the roster form.

(v) For the relation $R'$ defined in (iv), draw an arrow diagram.

## ☑ Chapter 03
## Trigonometric Functions

Q01. Given that $\sin x = -\frac{5}{13}$, x lies in third quadrant.
Based on the above information, answer the following questions.
(i) Find the value of $\sin 2x$.
(ii) Find the value of $\cos 2x$.
(iii) Find the value of $\sin\frac{x}{2}$.
(iv) Determine the value of $\cos\frac{x}{2}$.
(v) Determine the value of $\tan\frac{x}{2}$.

Q02. After retirement, Mr Ravi Dutt Sharma purchased a farm house in the shape of quadrilateral ABCD with $\angle A = 90°, \angle B = 72°, \angle C = 108°$ and $\angle D = 90°$. Mr Sharma also purchased a horse and a cow. One day, he tied the horse with a rope at vertex B and observed that it describes an arc of length 88 m when it moves along a circular path keeping the rope tight.
Based on the above information, answer the following questions.
(i) What is radian measure of $\angle B$?
(ii) What is the length of rope?
(iii) What will be the length of arc described by horse if he doubles the rope length?
(iv) What will be the length of arc described by cow, if it is tied at vertex C with the rope of same length as horse?
(v) What is the ratio of area that horse can cover to that of cow with same length of rope?

## ☑ Chapter 04
## Complex Numbers

Q01. While solving a typical equation a student Ayesha finds that one of the roots of the equation is a complex number $z = \frac{1+2i}{1-3i}$. Help her to find the answer of following questions.
(i) Find the standard form of $z$.
(ii) If $z = 2x + (4-y)i$, then obtain values of x and y.
(iii) Write the conjugate of $z$.
(iv) What is the modulus of $z$?
(v) Mention the quadrant in which $z$ lies?

## ☑ Chapter 05
## Linear Inequations

Q01. A company produced cassettes; one cassette costs the company ₹30 and also an additional fixed cost of ₹26000 per week. The company sold each cassette at ₹43.
Let x be the number of cassettes produced and sold by the company in a week.
From the above information, answer the following questions.
(i) Find the cost function of the company.
(ii) Find the revenue function of the company.
(iii) Find the profit function of the company.
(iv) How many cassettes must be produced by the company in a week to realize some profit?
(v) If company incurred an additional cost of ₹3 on each cassette per week, then how many cassettes must be produced by the company in a week so that there is no profit no loss?

## ☑ Chapter 06
## Permutations & Combinations

Q01. A school administration decides to send some of its students of class XI to an educational tour. From a class of 25 students, 10 are to be chosen for the tour.

There are three friends - Rajesh, Shreya and Deepa - who decide that either all of them will join or none of them will join.

Based on the above information, answer the following questions.

(i) In how many ways can the students be chosen for this educational tour, if these three friends will join?

(ii) In how many ways can the students be chosen for this educational tour, if these three friends will not join?

(iii) In how many ways can the students be chosen for this educational tour?

(iv) Mr O.P. GUPTA, the Mathematics teacher of school puts some questions for these three students - with a condition that if any one of them answers correctly then, they may join this tour.
He asks them to find the number of words formed using all the letters of 'Rajesh'. Deepa answers it correctly. What could be her answer?

(v) Further the teacher asked all of them to find the number of words formed using all letters of 'Deepa'. What could be the correct answer?

Q06. Anish appears in an examination.

While reading the instructions, Anish observed that the question paper consists of 12 questions divided into two parts - Part I and Part II, containing 5 and 7 questions, respectively.

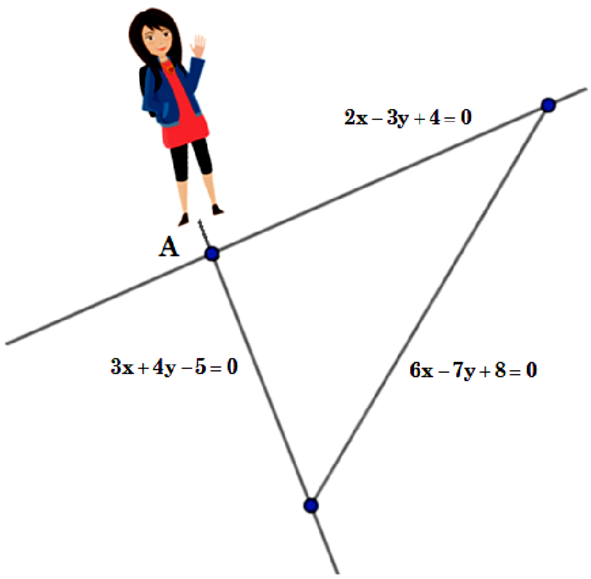
Based on the given information, answer the following questions

(i) If Anish is required to attempt 8 questions in all, selecting at least 3 from each part, then in how many ways can he select the questions?

(ii) If Anish is required to attempt 8 questions in all, selecting at most 3 from part I, then in how many ways can he select the questions?

## ☑ Chapter 09
## Straight Lines

Q01. Rajshri is standing at the junction (point A in the diagram shown below) of two straight paths represented by the equations $2x-3y+4=0$ and $3x+4y-5=0$.




Based on the given information, answer the following questions.

(i) Slope of the line $2x-3y+4=0$ is

(a) 2 (b) 3 (c) $-\frac{2}{3}$ (d) $\frac{2}{3}$

(ii) What is the x-intercept made by the line $3x+4y-5=0$?

(a) 3 (b) $\frac{5}{4}$ (c) $\frac{5}{3}$ (d) $\frac{3}{4}$

(iii) Coordinates of point A is

(a) $\left(\frac{1}{17},-\frac{22}{17}\right)$ (b) $\left(-\frac{1}{17},\frac{22}{17}\right)$ (c) $\left(\frac{1}{17},\frac{22}{17}\right)$ (d) $\left(-\frac{1}{17},-\frac{22}{17}\right)$

(iv) Rajshri wants to reach the path whose equation is $6x-7y+8=0$ in the least time. Then from the point A she must walk along a line which is

(a) perpendicular to the line $6x-7y+8=0$

(b) parallel to the line $6x-7y+8=0$

(c) not necessarily perpendicular to the line $6x-7y+8=0$

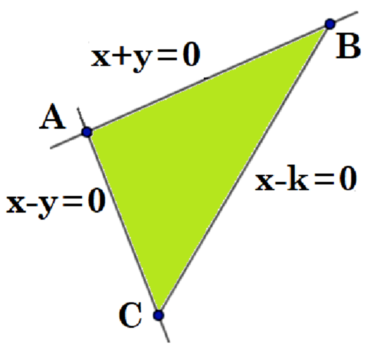
(d) not necessarily parallel to the line $6x-7y+8=0$

(v) The equation of the line along which she walks to reach the line $6x-7y+8=0$ in least time, is

(a) $102x+119y=125$ (b) $119x+102y=125$

(c) $109x+112y=125$ (d) $119x+102y=215$

Q02. A piece of land owned by a farmer is triangular in shape. See the figure given below.



The sides of the field are represented by $AB: x + y = 0,\ BC: x - k = 0,\ CA: x - y = 0$.

Based on the given information, answer the following questions.

(i) Vertex A is

(a) (k, k) (b) (0, 0) (c) (k, –k) (d) (–k, –k)

(ii) Coordinates of vertex B is

(a) (k, k) (b) (k, 0) (c) (k, –k) (d) (–k, –k)

(iii) Coordinates of vertex C is

(a) (k, k) (b) (0, 0) (c) (k, –k) (d) (–k, –k)

(iv) Area of the triangular field (ABC) is

(a) k Sq. units (b) $k^2$ Sq. units (c) $\frac{1}{2}k^2$ Sq. units (d) $\frac{1}{2}k$ Sq. units

(v) For the triangle ABC, which of the sides are perpendicular to each other?

(a) AB and BC (b) BC and CA

(c) AB and AC (d) None of the sides are perpendicular

## ☑ Chapter 10
## Conic Sections

Q01. A beam is supported at its ends by supports which are 12 m apart. Since the load is concentrated at its centre, there is a deflection of 3 cm at the centre and the deflected beam is in the shape of a parabola.

Based on the above information, answer the following questions.

(i) How far from the centre is deflection of 1 cm?

(ii) What will be the equation of parabola?

(iii) At a distance of 2 m from the centre, what will be the deflection of the beam?

(iv) What is the length of latus rectum of the parabola?

(v) What is the difference of deflection of beam at a distance of 1 m and 2 m from the centre?

## ☑ Chapter 12
## Limits & Derivatives

Q01. Mr Pardeep has a rectangular plot, which is used for growing vegetables.
Perimeter of plot is 50 m. Length and width of plot are x m and y m respectively.
Based on the above information, answer the following questions.

(i) Relation between x and y is

(a) $x + y = 50$ (b) $x + y = 100$ (c) $x + y = 25$ (d) $x = y$

(ii) Area function, $A(x) =$

(a) $x^2 - 5$ (b) $25x - x^2$ (c) $x^2 - 25x$ (d) $25 - x$

(iii) Derivative of $A(x)$ w.r.t. $x$, $A'(x) =$

(a) $2x$ (b) $-2x$ (c) $25 - 2x$ (d) $2x - 25$

(iv) Value of $x$ for which $A'(x) = 0$ is

(a) 25 (b) 12.5 (c) 5 (d) 0

(v) Value of $A'(x)$ at $x = 12.5$ is

(a) 156.25 (b) 250 (c) 0 (d) 144.25

## ☑ Chapter 14
## Probability

Q01. Two candidates Anil and Ashima appeared in a written test for a job position in a company.

The probability that Anil will qualify the test is 0.05 and that Ashima will qualify the test is 0.10.

The probability that both will qualify the test is 0.02.

Based on the given information, answer the following questions.

(i) Find the probability that both Anil and Ashima will not qualify the test.

(ii) Determine the probability that only one of the candidates will qualify the test.

Q02. On a week-end curfew due to Covid-19 pandemic, Soniya and Isha could not go nearest mall to have fun. They decided to involve themselves in various indoor activities which included playing with cards as well, apart from some other activities.

The pack of playing cards has a total of 52 cards.

Based on the given information, answer the following questions.

(i) If Soniya draws four cards from the pack of 52 playing cards, then what is the probability of getting three diamonds and one spade?

(ii) Isha took two cards from the pack. What is the probability of getting both cards of king?

# ANSWERS
## (MATHMISSION FOR XI)

## CHAPTER 01

### Exercise 1.1

Q01. (a) Not a set (b) Not a set (c) A set (d) Not a set (e) Not a set

Q02. (a) $\{0,1,2,3,4,5,6,7\}$ (b) $\{-2,1\}$ (c) $\{C, O, M, B, I, N, A, T, S\}$

(d) $\{3,5,7,9,11\}$ (e) $\{1,2,3,4,5,6,7\}$ (f) $\{1,2,3,...,9\}$

(g) $\{18, 27, 36, 45, 54, 63, 72, 81, 90\}$ (h) $\{a,e,i,o\}$

(i) $\left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}\right\}$ (j) $\{2, 3, 5\}$

Q03. (a) $\{x : x \text{ is an integer and } -4 \le x \le 5\}$

(b) $\{x : x = n^2 - 1, \text{ where n is a natural number and } 1 \le n \le 6\}$

(c) $\left\{x : x = \frac{n}{n+1}, \text{ where n is a natural number and } 1 \le n \le 6\right\}$

(d) $\{x : x = n^3 + n, n \in Z - Z^-, n \le 5\}$ (e) $\{x : x \text{ is a root of } x^2 - 4 = 0\}$

(f) $\{x : x = 7m, \text{ where } m \in N, 1 < m \le 12\}$ or $\{x : x = 7n, n \in N \text{ and } 7 < x < 90\}$

(g) $\{x : x \text{ is a prime number and } 50 < x < 100\}$

(h) $\{x : x \in N, x \text{ is equal to 1 or multiple of } 5\}$ (i) $\left\{x : x = \frac{1}{n^2}, n \in N\right\}$

(j) $\{x : x^2 \le 4, x \in Z\}$ (k) $\{x : x^2 \le 9, x \in N\}$

(l) $\{x : x = 3^n, n \in N, n \le 5\}$

Q04. i – c, ii – a, iii – b, iv – d Q05. $\in, \notin, \in, \notin$

Q06. (a) $\{1, 8, 27, 125\}$ (b) $\{-3, -2, -1, 0, 1, 2, 3\}$ (c) $\{a, e, i, o\}$ (d) $\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}\right\}$

Q07. (a) $\{\ \}$ or $\phi$ (b) $\{x : x \in T\}$ or $\{x : x \text{ is real and irrational number}\}$

Q08. $\{4,5,6,7\}$ Q09. $\{2m+1 : m \ge 0, m \in Z\}$

### Exercise 1.2

Q01. (a) Singleton set (b) Not a singleton set Q02. No pair is equal

Q03. (a) $A \ne B$ (b) $A = B$

Q04. (a) $\{x : x \in R, -5 < x \le 9\}$ (b) $[-3, 7]$ (c) $\{x : x \in R, a < x < a\}$

(d) $2^3 = 8$ (e) $\phi, \{5\}, \{6\}, \{5,6\}$

Q05. (a) $\{x : x \in R, -3 < x < 0\}$ (b) $\{x : x \in R, 6 \le x \le 12\}$

(c) $\left\{x : x \in R, -\frac{5}{2} \le x < 5\right\}$ (d) $\{x : x \in R, 1 < x \le 3\}$

Q06. (a) $\phi, \{a\}, \{b\}, \{a,b\}$ (b) $\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$

(c) $\phi$ (d) $\phi, \{-1\}, \{0\}, \{1\} \{-1, 0\}, \{0, 1\}, \{1, -1\}, \{-1, 0, 1\}$

Q07. $\phi$, {M}, {O}, {R}, {M, O}, {O, R}, {R, M}, {M, O, R}

Q08. $\phi$, {I}, {S}, {W}, {I, S}, {S, W}, {W, I}, {I, S, W}; 7

Q09. (a) $\in$ (b) $\notin$ (c) $\subset$ (d) = (e) = (f) $\not\subset$ (g) $\subset$ (h) $\subset$ (i) $\notin$ (j) $\notin$ (k) $\notin$

Q10. (a) Infinite set (b) Finite set (c) Infinite set (d) Infinite set

Q11. (a) Empty set (b) Non empty set (c) Null set (d) Null set

Q12. As $A \subset B$ so, B is a superset of A Q13. $m = 8,\ n = 4$ Q14. $\{10\}$

## Exercise 1.3

Q01. (a) $\{7, 9, 11\}$ (b) $\{7, 9, 11\}$ Q02. (a) {2} (b) {5, 7, 9, 11}

Q03. Set of rational numbers Q04. Only (c) is disjoint

Q05. $A' = \{1, 4, 5, 6\}$, $B' = \{1, 2, 6\}$, $A' \cap B' = \{1, 6\}$, $A \cup B = \{2, 3, 4, 5\}$ Q08. $\{2, 3, 4, 5, 6, 7\}$, $\{4, 5\}$

Q09. $A = \{c, d, e\}$ Q10. $X \cup Y = \{D, E, H, I, L, O\}$, $Y - X = \{O\}$

Q11. (a) $(-\infty, \infty) - [0, 3]$ or, $(-\infty, 0) \cup (3, \infty)$ (b) [0, 6] (c) [2, 3] (d) [0, 2)

Q12. $A'$ = the set of all boys of XI class Q13. 21N

Q14. (a) $\{x : x \in N \text{ and } x \neq 3\}$ (b) $\{x : x \in N$ and x is not a perfect square$\}$

## Exercise 1.4

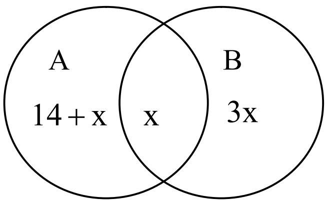
Q08. Let A = {0, 1}, B = {1, 2}, and C = {2, 0}. Accordingly, A ∩ B = {1}, B ∩ C = {2}, and A ∩ C = {0}. So A ∩ B, B ∩ C, and A ∩ C are non-empty. However it is clear that, $A \cap B \cap C = \phi$.

## Exercise 1.5

Q01. 29% Q02. 170

Q03. (a) 20 people can speak both Hindi and English (b) 480 people can speak Hindi only.

Q04.




; $x = 7$

Q05. 90, 20, 140, 60 Q06. 43

Q07. 62, 39, 1 Q08. 225

Q09. (a) 40 (b) 50 Q10. 4, 13

Q11. 60, 35, 13; 22 Q12. 9, 6

Q13. 300 Q14. 5, 4, 2, 1, 6, 11, 23, 2 Q15. 20%, 45%, 4%

Q16. 33%, 14%, 40%, 6%, 52%, 2%, 60% Q17. 215, 50

Q18. 20, 325 Q19. 24 Q20. (a) 35 (b) 11 (c) 11 Q21. 250

Q22. 23 Q23. $\{2, 3, 5\}$

# CHAPTER 02

## Exercise 2.1

Q01. (a) $m = 3,\ n = -1$ (b) $x = 2,\ y = 1$ (c) $a = 2,\ b = -3$

Q02. $x = 3,\ y = 2$ Q03. 15, 4 Q04. 6, 6, 9

Q07. $\{(a, 1), (a, 2), (a, 5), (b, 2)(b, 5), (b, 1)\}$ Q08. $A = \{x, y, z\}, B = \{1, 2\}$

Q09. $A = \{-1, 0, 1\}$, remaining elements of $A \times A$ are $(-1, -1), (-1, 1), (0, -1), (0, 0), (1, -1), (1, 0), (1, 1)$

Q10. $A \times A \times A = \{(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (-1, 1, 1), (1, -1, -1), (1, -1, 1), (1, 1, -1), (1, 1, 1)\}$

Q11. $R \times R = \{(x, y) : x, y \in R\}$ represents the coordinates of all the points in two dimensional space. Also $R \times R \times R = \{(x, y, z) : x, y, z \in R\}$ represents the coordinates of all the points in three dimensional space.

Q12. $\{(1, 4), (2, 4), (3, 4)\}$; $\{(1, 4), (2, 4), (3, 4)\}$;
$\{(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)\}$ and
$\{(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)\}$

Q13. $A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$; $A \times B$ will have $2^4 = 16$ subsets; these subsets are given as $\phi$,

$\{(1,3)\}, \{(1,4)\}, \{(2,3)\}, \{(2,4)\}, \{(1,3),(1,4)\}, \{(1,3),(2,3)\}, \{(1,3),(2,4)\}, \{(1,4),(2,3)\}, \{(1,4),(2,4)\}, \{(2,3),(2,4)\}, \{(1,3),(1,4),(2,3)\}, \{(1,3),(1,4),(2,4)\}, \{(1,4),(2,3),(2,4)\}, \{(1,3),(2,3),(2,4)\}$ and $\{(1,3),(1,4),(2,3),(2,4)\}$

Q14. (1, 5), (2, 3), (3, 5)

Q15. $\{(1,4,6),(1,4,8),(1,5,6),(1,5,8),(2,4,6),(2,4,8),(2,5,6),(2,5,8)\}$

## Exercise 2.2

Q01. $2^{m\times n}$ Q02. $2^6 = 64, 2^9 = 512$

Q03. $R = \{(1,3),(2,6),(3,9),(4,12)\}$; Domain of $R = \{1,2,3,4\}$; co-domain of $R = \{1,2,....,14\}$; and the Range of $R = \{3,6,9,12\}$

Q04. (a) $R = \{(1,1),(1,2),(1,3),(1,4),(1,6),(2,2),(2,4),(2,6),(3,3),(3,6),(4,4),(6,6)\}$
(b) Domain of $R = \{1,2,3,4,6\}$ (c) Range of $R = \{1,2,3,4,6\}$

Q05. $R = \{(1,4),(1,6),(2,9),(3,4),(3,6),(5,4),(5,6)\}$; Dom.$(R) = \{1,2,3,5\}$;
Codomain of $R = \{4,6,9\}$ and the Range of $R = \{4,6,9\}$

Q06. $\{(x,y): x$ is the square of $y,\ x \in P,\ y \in Q,\ y \neq 1\}$; $\{(9,3),(9,-3),(4,2),(4,-2),(25,5),(25,-5)\}$;
Domain $= \{4,9,25\}$ and the Range $= \{-2,2,-3,3,-5,5\}$

Q08. (a) Not true (b) Not true (c) Not true

Q09. $R = \{(2,2),(2,4),(2,6),(2,8),(4,4),(4,8)\}$; Dom.$(R) = \{2,4\}$, Range$(R) = \{2,4,6,8\}$

Q10. $R = \{(1,1),(2,8),(3,27),(4,64)\}$; Domain = {1, 2, 3, 4} and Range = {1, 8, 27, 64}

Q11. $R = \{(-3,4),(-2,3),(-1,2),(0,1),(1,0),(2,1),(3,2)\}$; Domain of R = {–3, –2, –1, 0, 1, 2, 3} and Range of R = {0, 1, 2, 3, 4} Q12. $\because R \subseteq A \times B$ so, R is a relation from A to B

Q13. (a) See in the **SOLUTIONS OF MATHMISSION** book (b) {1,2,3,4}
(c) {1,4,9,16} (d) {1,4,9,16,25}

Q14. Domain of R = Z, Range of R = Z Q15. (a) {1, 2, 3, 4} (b) {±1,±2,±4}

Q16. Domain of $R = \{1,3,5,7,...,39\}$; range of $R = \{1,2,3,4,...,20\}$

## Exercise 2.3

Q01. 2.1 Q03. $x^2+2x+1;\ x^2-2x-1;\ 2x^3+x^2;\ \dfrac{x^2}{2x+1}, x \neq -\dfrac{1}{2}$

Q04. (a) 0 (b) –4 (c) –1 (d) 0 Q06. $a = 2, b = -1$

Q07. $f(x) = 2x-1$ Q08. $a = \dfrac{4}{3}, b = 10$ Q09. Range of f = {3, 5, 11, 13}

Q10. Since the first components of ordered pairs belonging to g are 2, 4, 6, 8, 10, 12, 14 which are different and have distinct images i.e., different second components of ordered pairs hence, g is a function.

Q11. (a) R – {2, 6} (b) R – {1, 4} (c) R – {2, 3}

Q12. (a) R is not a function because element 4 has two distinct images.
(b) R is not a function because element 2 does not have unique image.
(c) R is a function because every element has a unique image.

Q13. (a) This arrow diagram represents a function because every element has a unique image.
(b) This arrow diagram does not represent a function as element 2 has two distinct images.

Q14. (i) $R, [2,\infty)$ (ii) $[1,\infty), [0,\infty)$ (iii) $[-3,3], [0,3]$
(iv) R, Z (v) $R, (-\infty, 0]$ (vi) R, non-negative real numbers

Q15. Any positive real number such that $0 \leq y < 1$ where $y = f(x)$ i.e., $[0,1)$

Q16. (a) {4, 6, 9, 10} (b) R (c) $R-\{-\sqrt{3},\sqrt{3}\}$ (d) [–2, 2]

(e) $\left[-1-\sqrt{3},-1+\sqrt{3}\right]$ (f) $\left(-\infty,\dfrac{7-\sqrt{17}}{2}\right]\cup\left[\dfrac{7+\sqrt{17}}{2},\infty\right)$

(g) (–1, 1) (h) $(-\infty,1)\cup(2,\infty)$ i.e., $]-\infty,1[\cup]2,\infty[$ (i) $(-\infty,\infty)$

(j) $\{x:x\in Z,x\geq 0\}$ (k) [–3, 3] (l) R (m) R–{–1, 1}

(n) R–{1} (o) $(-\infty,\infty)$ (p) [0, 1] (q) R–{0}

(r) $(3,\infty)$ (s) $[-2,0)\cup(0,1)$ (t) R (u) Z

(v) $(0,\infty)$ (w) R–Z (x) $(-\infty,0)$ (y) $\phi$

(z) $(0,\infty)$

Q17. (a) $\{1,2,3\}$ (b) $\{-1,0,3,8\}$ (c) $\{5,7,9,11,...\}$ (d) $y\in\left[-\dfrac{1}{2},\dfrac{1}{2}\right]$

(e) $[0,1)$ i.e., $0\leq y<1$ (f) $(-\infty,0)\cup[1,\infty)$ i.e., $y<0$ or $y\geq 1$

(g) $R-\{2\}$ (h) $\left[\dfrac{1}{3},1\right]$ (i) $[0,3]$ (j) $(-\infty,0]$

(k) $[0,\infty)$ (l) $(-\infty,0)\cup\left[\dfrac{1}{3},\infty\right)$ (m) [0, 2] (n) {1, 3}

(o) $(0,\infty)$ (p) {–1} (q) {–1, 1} (r) (–∞, 1]

(s) {1, 2, 3} (t) $\{x!:x\in Z,x\geq 0\}$

Q18. (a) Dom. : R, Range : $[-3,\infty)$ (b) Dom.: R, Range: $(-\infty,1]$ (c) Dom.: $R-4$, Range: $\{-1\}$

Q19. We know that a relation f from a set A to a set B is said to be a function if every element of set A has unique images in set B. Since 2, 6, –2, –6 ∈ **Z**, (2 × 6, 2 + 6), (–2 × –6, –2 + (–6)) ∈ f i.e., (12, 8), (12, –8) ∈ f. It can be seen that the same first element i.e., 12 corresponds to two different images i.e., 8 and –8. Thus, relation f is not a function. Q20. R

Q21. (a) $\because A\times B=$ {(1, 1), (1, 5), (1, 9), (1, 11), (1, 15), (1, 16), (2, 1), (2, 5), (2, 9), (2, 11), (2, 15), (2, 16), (3, 1), (3, 5), (3, 9), (3, 11), (3, 15), (3, 16), (4, 1), (4, 5), (4, 9), (4, 11), (4, 15), (4, 16)}. And $f=\{(1,5),(2,9),(3,1),(4,5),(2,11)\}$. As $f\subset A\times B$, so it is a relation from A to B.

(b) As no element of A should have more than one image in B. Here the element 2 corresponds to two different images i.e., 9 and 11. So, f is not a function from A to B.

Q24. Domain : R, Range : {2} Q25. (a) $(-\infty,-2)\cup[4,\infty)$ (b) $[1,2]$

Q26. $(-\infty,-1)\cup(1,4]$ Q28. 0, 1 Q29. $\left\{-2,\dfrac{1}{2}\right\}$

Q30. $f(x)=\begin{cases}-2x, & -3\leq x<-2\\ 4, & -2\leq x<2\\ 2x, & 2\leq x\leq 3\end{cases}$ Q34. [2, 4)

## Exercise 2.4

Q01. $ln(ln\ x)$ Q02. True only when $x\in R^{+}$ Q03. $\sqrt{e}$ Q04. $\dfrac{\log_e(\log x)}{\log_e 5}$

Q05. $x^5$ Q06. $\dfrac{1}{7}$ Q07. 1 Q08. 2 Q09. $\dfrac{1}{2}$

Q10. $x\in R-0$ Q11. $x\in R-\{0\}$ Q12. $x\in(3,\infty)$ Q13. $x\in(-\infty,1)-\{0\}$

Q14. $x \in \left(0, \frac{1}{3}\right)$.

# CHAPTER 03

## Exercise 3.1

Q01. (a) $\frac{25\pi}{18}$ (b) $-\frac{19\pi}{72}$ (c) $\frac{5\pi}{36}$ (d) $\frac{16321\pi}{64800}$ or 0.79158 (Approx.) (e) $\frac{9\pi}{20}$

Q02. (a) $39^\circ 22' 30''$ (b) $-229^\circ 5' 27''$ (Approx.) (c) $343^\circ 38' 11''$ (Approx.) (d) $210^\circ$
(e) $68^\circ 43' 37''$

Q03. $12^\circ 36'$

Q04. $\frac{20\pi}{3}$ cm or 20.952 cm (Approx.) Q05. $22:13$ Q06. 6.28 cm

Q07. $30^\circ, 60^\circ, 90^\circ$ Q08. $81^\circ, 9^\circ$ Q09. $51^\circ, 39^\circ$ Q10. $7^\circ$

Q11. 1.536 m Q12. $\frac{4\pi}{5}$ Q13. $100^\circ$ Q14. $127^\circ 30'$

Q15. $\frac{\pi}{4}, 75^\circ$ Q16. 70 m

Q17. $\cos x = -\frac{1}{5}, \sec x = -5, \operatorname{cosec} x = -\frac{5}{2\sqrt{6}}, \tan x = 2\sqrt{6}, \cot x = \frac{1}{2\sqrt{6}}$

Q18. $\sin x = \frac{12}{13}, \cos x = -\frac{5}{13}, \tan x = -\frac{12}{5}, \operatorname{cosec} x = \frac{13}{12}, \sec x = -\frac{13}{5}$

Q19. $\frac{\sqrt{3}}{2}, -\sqrt{3}, -\sqrt{2}, 2$ Q20. $-2, \frac{1}{\sqrt{2}}$ Q21. 1 Q22. $-1$ Q23. $14^\circ 19' 5''$

## Exercise 3.2

Q01. $\frac{\sqrt{3}-1}{2\sqrt{2}}, \frac{\sqrt{3}-1}{2\sqrt{2}}, 2-\sqrt{3}, 2+\sqrt{3}, \frac{\sqrt{3}+1}{2\sqrt{2}}, \frac{1+\sqrt{3}}{1-\sqrt{3}}$ Q02. $2-\sqrt{3}, \sqrt{2}-1, \frac{\sqrt{3}-1}{2\sqrt{2}}, \frac{\sqrt{2-\sqrt{2}}}{2}$

Q03. $\sin 6x - \sin 2x$ Q04. $2\sin 8\theta \cos 4\theta$ Q05. $-\frac{56}{65}, \frac{33}{65}, -\frac{16}{63}$

Q06. $\frac{220}{221}, \frac{171}{221}, \frac{220}{21}$ Q07. $\frac{2}{11}$ Q08. $-\frac{120}{169}, -\frac{120}{119}$ Q09. $\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 2$

Q10. (a) $\frac{\sqrt{6}}{3}, \frac{\sqrt{3}}{3}, \sqrt{2}$ (b) $\frac{\sqrt{8+2\sqrt{15}}}{4}, -\frac{\sqrt{8-2\sqrt{15}}}{4}, -[4+\sqrt{15}]$ (c) $\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}, 3$

Q11. $\frac{1}{\sqrt{3}}$ Q12. $45^\circ$

## Exercise 3.3

Q06. 0 Q16. $\sqrt{3}$ Q17. (a) $\frac{2ab}{a^2+b^2}$

## Exercise 3.4

Q01. (a) $\frac{4\pi}{3}, \frac{5\pi}{3}; n\pi + (-1)^n \frac{4\pi}{3}, n \in Z$ (b) $\frac{7\pi}{6}, \frac{11\pi}{6}; n\pi + (-1)^n \frac{7\pi}{6}, n \in Z$
(c) $\frac{\pi}{3}, \frac{4\pi}{3}; n\pi + \frac{\pi}{3}, n \in Z$

Q02. (a) $2n\pi+\frac{\pi}{2}, 2m\pi-\pi; m,n\in Z$ (b) $2n\pi+\frac{\pi}{6}, 2m\pi-\frac{\pi}{6}; m,, n\in Z$ (c) $n\pi+\frac{3\pi}{4}, n\in Z$

(d) $(2n+1)\frac{\pi}{4}, 2m\pi\pm\frac{\pi}{3}; m,n\in Z$ (e) $\frac{n\pi}{3}, m\pi\pm\frac{\pi}{3}; m,n\in Z$ (f) $\frac{n\pi}{2}, \frac{m\pi}{2}+\frac{3\pi}{8}; m,n\in Z$

(g) $n\pi+(-1)^n\frac{7\pi}{6}, (2m\pm1)\frac{\pi}{2}; m,n\in Z$ (h) $n\pi+(-1)^n\frac{7\pi}{6}, n\in Z$ (i) $n\pi+\frac{5\pi}{6}, n\in Z$

(j) $2n\pi+\frac{\pi}{6}, 2m\pi-\frac{\pi}{2}; m,n\in Z$ (k) $2n\pi\pm\frac{\pi}{3}, n\in Z$

(l) $x=\frac{n\pi}{3}, n\pi\pm\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$, where $n\in Z$ (m) $\theta=(2n+1)\frac{\pi}{8}, n\pi\pm\frac{\pi}{3}$ where $n\in Z$

(n) $\theta=-\frac{n\pi}{12}$ or $\theta=\frac{m\pi}{12}$, where $m\in Z$. Q03. $2n\pi\pm\frac{5\pi}{6}, n\in Z$ Q04. $\frac{1}{7}\left(n\pi+\frac{\pi}{2}\right), n\in Z$

Q05. (a) $n\pi+(-1)^n\frac{7\pi}{6}, 2n\pi\pm\frac{2\pi}{3}, n\in Z$ (b) $n\pi\pm\frac{\pi}{3}, n\pi, n\in Z$ (c) $n\pi\pm\frac{\pi}{6}, n\pi, n\in Z$

(d) $2n\pi, n\pi, n\in Z$ (e) $(3n+1)\frac{\pi}{9}, n\in Z$

(f) $2n\pi\pm\frac{\pi}{3}, 2n\pi\pm\frac{2\pi}{3}, (2n+1)\frac{\pi}{2}, n\in Z$ (g) $2n\pi+\frac{5\pi}{12}, 2n\pi-\frac{\pi}{4}, n\in Z$

(h) $2n\pi+\frac{\pi}{2}, 2n\pi-\frac{\pi}{6}, n\in Z$

## Exercise 3.5

Q01. $\frac{\pi}{20}$ Q02. $A\in\left[\frac{3}{4},1\right]$ Q03. $x\in R, \left[\frac{1}{7},\frac{1}{3}\right]$

Q04. Domain $= x\in R$, Range $=[-3,3]$ Q05. (a) Positive (b) Negative

Q06. 10, –4 Q07. 0 Q08. $\tan x$

Q09. 4 Q10. (b) $\frac{\pi}{4}, \frac{3\pi}{4}$

## Exercise 3.6

Q01. 0 Q02. 0 Q03. $\frac{7\pi}{24}$ Q05. $\frac{2\times(n-m)(1+nm)}{(1-m^2)(1-n^2)+4nm}$

Q08. $x=2n\pi\pm\pi, n\in Z$ Q09. 0 Q10. $\theta=(2n+1)\frac{\pi}{6}, (2n-1)\frac{\pi}{4}\ \forall\ n\in Z$

Q11. $-\frac{1}{16}$ Q13. $\frac{3}{2}$ Q16. $x=\frac{\pi}{3}, \frac{2\pi}{3}, \frac{\pi}{6}, \frac{5\pi}{6}$

Q17. $\frac{1}{a}-\frac{1}{b}$ Q18. Number of solutions are 3 Q19. $\frac{\sqrt{5}+1}{8}$

Q20. 10, 8 Q21. $\pi-2$ radians Q22. $130^\circ$

Q23. (b) $\pm1$ (d) Not possible (e) $\frac{1}{\sqrt{2}}$ (g) $x=\frac{\pi}{4}$ (h) $\cos x=\frac{\sqrt{1+\sqrt{5}}}{2}$

Q24. (i) $2\sin\left(x-\frac{\pi}{6}\right)$ (ii) $-2\cos\left(x+\frac{\pi}{3}\right)$ Q25. $R=2, x=\frac{\pi}{6}$

Q26. $x=\frac{n\pi}{2}+(-1)^n\frac{\pi}{4}\ \forall\ n\in Z$.

# CHAPTER 04

## Exercise 4.1

Q01. $3-4i$ Q02. $13+0i$ Q03. $1+4i$ Q04. $-4+0i$

Q05. $-\frac{2}{25}+\frac{14}{25}i$ Q06. $1+0i$ Q07. $0-i$ Q08. $\frac{7}{25}+\frac{24}{25}i$

Q09. $0+0i$ Q10. $\frac{1}{4}+\frac{9}{4}i$ Q11. $0-\frac{8}{25}i$ Q12. $0+\frac{1}{256}i$

Q13. $0-\frac{7\sqrt{2}}{2}i$ Q14. $2-2i$ Q15. $\frac{307}{442}+\frac{599}{442}i$ Q16. $x=\frac{5}{13}, y=\frac{14}{13}$

Q17. $x=\frac{14}{15}, y=\frac{1}{5}$ Q18. $x=\frac{16}{5}, y=\frac{13}{5}$ Q19. $x=3,\ y=-1$

Q20. $\frac{3}{13}-\frac{2}{13}i$ Q21. $\frac{\sqrt{3}}{5}+\frac{\sqrt{2}}{5}i$ Q22. $0+i$ Q23. $\frac{9}{10}+\frac{7}{10}i$; $\text{Re}=\frac{9}{10}$, $\text{Im}=\frac{7}{10}$

Q24. $-9-6i$

## Exercise 4.2

Q01. $\frac{1}{\sqrt{2}}, \frac{\pi}{4}, \frac{1}{2}-\frac{1}{2}i$

Q02. $\frac{\sqrt{5}}{6}$, $2\pi-\tan^{-1}2$, $\frac{1}{6}+\frac{1}{3}i$; [Also, the Principal value of $\arg z=(2\pi-\tan^{-1}2)-2\pi=-\tan^{-1}2$]

Q03. $2, -\frac{5\pi}{6}, -\sqrt{3}+i$ Q04. $\frac{1}{2}, \frac{5\pi}{6}, -\frac{\sqrt{3}}{4}-i\frac{1}{4}$ Q05. $2, \frac{7\pi}{4}\left(\text{or } -\frac{\pi}{4}\right), \sqrt{2}+\sqrt{2}i$

Q06. $\sqrt{6}, \frac{3\pi}{4}, -\sqrt{3}-\sqrt{3}i$ Q07. $\sqrt{10}, \frac{3\pi}{4}, -\sqrt{5}-\sqrt{5}i$ Q08. $2, \frac{11\pi}{6}\left(\text{or } -\frac{\pi}{6}\right), \sqrt{3}+i$

Q09. $\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$ Q10. $2\left(\cos\left(\frac{-2\pi}{3}\right)+i\sin\left(\frac{-2\pi}{3}\right)\right)$

Q11. $2\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$ Q12. $\sqrt{2}\left(\cos\left(-\frac{3\pi}{4}\right)+i\sin\left(-\frac{3\pi}{4}\right)\right)$

Q13. $\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right)$ Q14. $1(\cos\pi+i\sin\pi)$ Q15. $1\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$

Q16. $2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$ Q17. $\frac{1}{2}\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$ Q18. $8\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$

Q19. $\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$ Q20. $\sqrt{2}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$

Q21. $\frac{63}{25}+\frac{16}{25}i$ Q22. $\frac{\pi}{2}$ Q23. $-2\sqrt{3}+2i$

Q24. $6\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$ Q25. $2, -\frac{2\pi}{3}$ Q26. $\sqrt{2}+\sqrt{2}i$

Q27. $\sqrt{5}\left(\cos\left(\tan^{-1}\left(\frac{1}{2}\right)-\pi\right)+i\sin\left(\tan^{-1}\left(\frac{1}{2}\right)-\pi\right)\right)$

Q28. (a) $1\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$ (b) $2(\cos 0+i\sin 0)$ Q29. 0 Q30. (i) 1 (ii) 1

## Exercise 4.3

Q01. (a) $\theta=n\pi,\ n\in Z$ (b) $\theta=m\pi+(-1)^m\frac{4\pi}{3},\ n\pi+(-1)^n\frac{\pi}{3};\ m,\ n\in Z$

Q02. $\theta=(2n+1)\frac{\pi}{2},\ n\in Z$ Q03. $\text{Re}\left(\frac{z_1z_2}{\overline{z_1}}\right)=4, \text{Im}\left(\frac{z_1z_2}{\overline{z_1}}\right)=2$ Q11. 4

Q12. $\sqrt{2}$ Q13. 2 Q15. $|z_1|=3$ Q16. 6

Q17. (a) $\cos n\theta+i\sin n\theta$ (b) $\sin\theta+i\cos\theta$ Q19. 1 Q24. $\frac{3}{4}+i$

Q25. $0+i0$ Q27. $a-ib$ Q28. $\frac{5\pi}{12}+0$ Q29. 1:3 Q30. $x-y+1=0$

Q33. Infinite number of solutions

## Exercise 4.4

Q01 $2\pm 3i$ Q02. $0-4i,\ 0+3i$ Q03. $\frac{-1\pm\sqrt{7}i}{2\sqrt{2}}$ Q04. $\pm i2\sqrt{3}$

Q05. $-\frac{1}{4}\pm i\frac{\sqrt{7}}{4}$ Q06. $-\frac{\sqrt{2}}{4}\pm i\frac{\sqrt{14}}{4}$ Q07. $-\frac{1}{2}\pm i\frac{\sqrt{2\sqrt{2}-1}}{2}$ Q08. $\frac{1}{2}\mp i\frac{\sqrt{7}}{2}$

# CHAPTER 05

## Exercise 5.1

Q01. $\{1,2,3,4,5,6\};\{\ldots-3,-2,-1,0,1,2,3,4,5,6\}$ Q02. $(-\infty,2),\{-2,-1,0,1\}$

Q03. $x\le -3$ Q04. $x\ge 4$ Q05. $x\le 120$ Q06. $x>4$ Q07. $x\ge 3$

Q08. $x<-50$ Q09. $-2\le x\le 4$ Q10. $-\frac{34}{3}\le x\le\frac{22}{3}$ Q11. $(-\infty,-3]\cup[7,\infty)$

Q12. $x\ge -\frac{2}{7}$ Q13. $(-\infty,2]$ Q14. $x\ge 1$ Q15. $\left(\frac{4}{103},\infty\right)$

Q16. $x>3$ Q17. No solution Q18. $2\le x<6$ Q19. $(5,\infty)$

Q20. $\left[-7,\frac{13}{2}\right]$ Q21. $x\in(-3,0)$

## Exercise 5.2

Q01. Between 52 and 77 Q02. (11,13), (13,15), (15,17) Q03. 9 cm

Q04. Greater than or equal to 8 but, less than or equal to 22

Q05. More than 120 litres but less than 300 litres

Q06. More than 320 litres but less than 1280 litres

Q07. More than 562.5 litres but less than 900 litres

Q08. Between 9.8 km and 13.8 km Q09. 86 °F and 95 °F

Q10. $5x+y\le 100,\ x+y\le 60,\ x\ge 0,\ y\ge 0$ where x : no. of tables and y : no. of stools

Q11. More than 2000 toys

## Exercise 5.3

Q01. $x<2$ Q02. $(2,6)$ Q03. $(-\infty,2)\cup(5,\infty)$ Q04. $(-\infty,-5)\cup(5,\infty)$

Q05. $(-\infty,-3)\cup(2,\infty)$ Q06. $(-\infty,3)$

Q07. (a) $(-\infty,1]\cup(3,\infty)$ (b) $\left(1,\frac{7}{4}\right]$ (c) $(-\infty,-1]\cup[7,\infty)$ (d) $(-\infty,-2)\cup[-1,1]\cup(2,\infty)$

(e) $\left[\frac{7}{6},\frac{3}{2}\right]$ (f) $\left(-\infty,-\frac{1}{2}\right]\cup\left[\frac{7}{2},\infty\right)$ (g) $(-\infty,0]\cup[4,\infty)$

(h) $x\in\left(-\infty,-\frac{2}{3}\right]\cup[4,\infty)$ (i) $x\in(-1,1)\cup(2,3)$

Q08. $x\in(-\infty,1)$

# CHAPTER 06

## Exercise 6.1

Q01. 5040 Q02. 108, 60 Q03. 336 Q04. 12 Q05. 320 Q06. 30240
Q07. 16 Q08. 100 Q09. 60 Q10. 180 Q11. 375 Q12. 64
Q13. 6 Q14. 10 Q15. 3 Q16. $\frac{9!}{4!3!2!}$
Q17. $3!\times(8!\times6!\times4!)$ Q18. 40320 Q19. 360, 720, 480, 120 Q20. $\frac{9!}{4!2!}$
Q21. $\frac{12!}{3!4!2!},\frac{11!}{3!2!4!},\frac{8!}{3!2!}.\frac{5!}{4!},\frac{12!}{3!4!2!}-\frac{8!}{3!2!}.\frac{5!}{4!},\frac{10!}{3!2!4!}$ Q22. $6!3!,\ 8!-6!3!$
Q23. 1814400, 2419200, 25401600 Q24. $\frac{9!}{3!2!}$ Q25. 33810
Q26. 151200 Q27. $\frac{10!}{3!2!}-1,\ \frac{9!}{3!2!}$ Q28. 144 Q29. $\frac{9!}{2!}$
Q30. 100 Q31. $89^{th}$ Q32. 120 Q33. $\frac{7!}{3!2!}-\frac{6!}{3!2!}$ Q34. 27216
Q35. 180 Q36. 120 Q37. 1440 Q38. (a) 36000 (b) 14400
Q39. 2880 Q40. 1693440 Q41. 56 Q42. PAANVR Q43. $\frac{5!}{2!}$, NAAIG
Q44. 907200 Q45. 5040 Q46. 48, 144

## Exercise 6.2

Q02. 3 Q03. 6 Q04. 2000 Q05. 3960 Q06. 210 Q07. 45
Q08. 120 Q09. ${}^{m}C_3$ Q10. ${}^{m}C_2-m$ Q11. 10 Q12. 10, 6
Q13. 504, 588, 1632 Q14. 270725 (a) 2860 (b) $13^4$ (c) 495 (d) 105625 (e) 29900
Q15. 420 Q16. ${}^{22}C_7+{}^{22}C_{10}$ Q17. 50400 Q18. 3600 Q19. 2880
Q20. 265 Q21. 66 Q22. (a) 91 (b) 435 Q23. 26
Q24. ${}^{27}C_2-{}^{13}C_2-{}^{11}C_2+2$ Q25. ${}^{12}C_4\times{}^{8}C_4\times{}^{4}C_4$ Q26. (a) 19 (b) $n=\frac{7}{2}$
Q27. 150 Q28. 1152 Q29. 24 Q30. $\frac{6!}{3!2!},\frac{5!}{3!2!}$ Q31. 72

## Exercise 6.3

Q01. HCF : 24, LCM : 720 Q02. $x=225$ Q03. $n=3$ Q04. 40320
Q05. 720 Q06. 288 Q07. 720 Q08. 1000 Q09. 277200
Q10. 60 Q11. 330 Q12. 6 Q13. 455 Q14. 5040
Q15. 450 Q16. 90 Q17. 2404080 Q18. (a) 90 (b) 185
Q19. 485 Q20. 2454 Q21. 817190 Q22. 28800 Q23. 120
Q25. 365 Q26. 12
Q27. 3905 (if digits are repeated) and 325 (if digits are not repeated); 3124 (if digits are repeated) and 260 (if digits are not repeated).

# CHAPTER 07

## Exercise 7.1

Q01. (a) $\frac{64}{x^6}-\frac{96}{x^4}+\frac{60}{x^2}-20+\frac{15x^2}{4}-\frac{3x^4}{8}+\frac{x^6}{64}$ (b) $x^{10}+15x^7+90x^4+270x+\frac{405}{x^2}+\frac{243}{x^5}$

Q02. 9509900499 Q03. $8(a^3b+ab^3),\ 40\sqrt{6}$ Q04. $2x(x^4+10x^2y^2+5y^4),\ 58\sqrt{2}$

Q05. $(1.01)^{1000000}$ Q06. $a=1$ Q07. (a) 672 (b) –1365

Q08. $5^{th}$ term i.e., $\frac{5}{12}$ Q09. (a) $\frac{35x^{12}}{8}$ (b) $\frac{14}{27}x^5y^4$ and $\frac{14}{9}x^4y^5$ (c) $-\frac{105}{8}x^{13}$ and $\frac{35}{48}x^{15}$

Q10. 18564 Q11. 14 Q12. $x=2, a=3, n=5$ Q13. 4

Q14. 7, 3 Q15. –438 Q16. 171 Q17. –6

Q18. ${}^nC_{n-r+1}x^{r-1}a^{n-r+1}$ Q19. $2a^8+12a^6-10a^4-4a^2+2$

Q20. $(-1)^5.{}^9C_5\left(\frac{x^3}{2}\right)^4\left(\frac{2}{x^2}\right)^5=-252x^2$ Q21. 10 Q22. ${}^{12}C_3(-3)^3x^3$

Q23. $\frac{{}^{18}C_9}{2^9}$ Q24. 55 Q25. 11, 7 Q26. $a=3, b=5, n=6$

Q27. (a) $\frac{16}{x}+\frac{8}{x^2}-\frac{32}{x^3}+\frac{16}{x^4}-4x+\frac{x^2}{2}+\frac{x^3}{2}+\frac{x^4}{16}-5$

(b) $27x^6-54ax^5+117a^2x^4-116a^3x^3+117a^4x^2-54a^5x+27a^6$

(c) $1-4x+10x^2-16x^3+19x^4-16x^5+10x^6-4x^7+x^8$ Q28. 21

Q29. (a) $\frac{n-r+1}{r}$ (b) Use ${}^nC_r+{}^nC_{r-1}={}^{n+1}C_r \Rightarrow {}^{2n+1}C_7$ Q30. 256 Q31. 10

## Exercise 7.2

Q20. 0 Q24. 7 or 14.

## Exercise 7.3

Q01. ${}^{10}C_5\,y^{5/2}x^{5/3}$ Q02. ${}^{31}C_6-{}^{21}C_6$ Q03. 210

# CHAPTER 08

## Exercise 8.1

Q01. 8, 11, 14, 17 Q02. 1 Q03. 0 Q05. 14

## Exercise 8.2

Q03. $r=\frac{5}{2}$ or $\frac{2}{5}$; Terms are $\frac{2}{5},1,\frac{5}{2}$ or $\frac{5}{2},1,\frac{2}{5}$ Q04. $r=\frac{2}{3}$ or $\frac{3}{2}$; Nos. are 9, 6, 4 or 4, 6, 9

Q05. 469 Q06. $r=2$ or $\frac{1}{2}$; Numbers are 8, 16, 32 or 32, 16, 8

Q07. 8, –4, 2 and 8 Q08. $r=2$ or $\frac{1}{2}$; Numbers are 3, 6, 12 or 12, 6, 3

Q09. $r=3$ or $\frac{1}{3}$; Numbers are 4, 12, 36 or 36, 12, 4 Q13. $rR$ Q16. 3

## Exercise 8.3

Q01. $2^{12}+31$ Q02. 4

Q07. (a) $\frac{8}{81}(10^{n+1}-10-9n)$ (b) $\frac{10}{9}(10^n-1)-n$ (c) $\frac{5}{9}\left(n-\frac{10^n-1}{9\times10^n}\right)$

Q08. 120, 480, $(30)(2)^n$ Q09. ₹$500(1.1)^{10}$ Q10. $n=7$

## Exercise 8.4

Q01. 4, 16 or 16, 4 Q02. $x^2-16x+25=0$ Q03. $\lambda=-\frac{1}{2}$

## Exercise 8.5

Q01. (a) $\frac{3}{2}$ (b) $\frac{3}{4}$ (c) 3 (d) 5.263 (e) $\frac{5}{12}$ (f) $-1$ (g) 0.75 (h) $\frac{4+3\sqrt{2}}{2}$ (i) $1+x$

Q02. 16 Q03. $\frac{2}{3}$ Q04. $1,\frac{1}{2},\frac{1}{4},\ldots$ Q05. $4,1,\frac{1}{4},\frac{1}{16},\ldots$ Q06. $10,5,\frac{5}{2},\ldots$

Q07. $\frac{1}{2}$ Q09. $5+\frac{10}{3}+\frac{20}{9}+\ldots$ Q10. $19,\frac{38}{3},\frac{76}{9},\ldots$ Q12. $\frac{419}{990}$

Q13. (a) $\frac{14}{90}$ (b) $\frac{712}{999}$ (c) $\frac{317}{90}$ Q14. $\frac{31}{45}$ Q16. 7

## Exercise 8.6

Q01. Rs.16680 Q02. Rs.39100 Q03. Rs.43690
Q04. Rs.17000, Rs.20000 Q05. Rs.5120 Q06. 25 days
Q07. 2, 5, 8 or 26, 5, –16 Q11. 753 Q12. –3, 1, 5, 25

## Exercise 8.7

Q01. $a_{20}=40\times42$

Q02. (a) $\frac{n(n+2)(n+4)}{3}$ (b) $\frac{n(4n^2+6n-1)}{3}$ (c) $\frac{3^n+8n-1}{2}$

(d) $\frac{n^2(n+1)}{2}$ (e) $\frac{n(n+1)(3n^2+5n+1)}{6}$ (f) $3n(n+1)(n+3)$

(g) $\frac{n(n+1)(n+2)}{6}$ (h) $\frac{n}{n+1}$ (i) $\frac{n}{n+1}$

(j) $\frac{n(n+1)(n+2)}{6}$ (k) $\frac{n}{2n+1}$ (l) $\frac{n}{3n+1}$

Q03. $S_{10}=1210$ Q04. $S_n=\frac{n(2n^2+9n+13)}{24}$

Q06. $S_n=\frac{n(n+1)(3n^2-n+4)}{12}$

Q08. (a) $S_n=2^{n+1}-n-2$ (b) $S_n=\frac{1-x^n}{(1-x)^2}-\frac{nx^n}{1-x}$

(c) $S_{100}=1+99\times2^{100}$ (d) $50(1-i)$

Q09. $S_n=\frac{n(n+1)(n+2)}{6}$ Q10. $S=\frac{70}{81}\left[269+\frac{1}{10^{30}}\right]$ Q11. $S_{20}=35720$

Q12. **When n is even :** $S=\frac{n(n+1)^2}{2}$; **when n is odd :** $S=\frac{n^2(n+1)}{2}$.

# CHAPTER 09

## Exercise 9.1

Q01. $\frac{121}{2}$ Sq. units Q02. $-\sqrt{3}$

Q03. (a) 1 and 2, or $\frac{1}{2}$ and 1; or $-1$ and $-2$, or $-\frac{1}{2}$ and $-1$ (b) 3 or $-\frac{1}{3}$

Q04. $(0,a),(0,-a)$ and $(-\sqrt{3}a,0)$ or $(0,a),(0,-a)$ and $(\sqrt{3}a,0)$ Q06. $-1$

## Exercise 9.2

Q01. $(2-\sqrt{3})x-y-4=0$ Q02. $\sqrt{3}x-y-4\sqrt{3}=0$ Q03. $2\sqrt{2}x+y-2(\sqrt{2}+1)=0$

Q04. $12x-5y=25$ Q05. $(2-\sqrt{3})x-y-4+2\sqrt{3}=0$ Q06. $x-y=2$

Q07. $2x+y-10=0$ Q08. $3x-8y=0;\ 3x-2y=0$ Q09. $3x-4y=3a,\ x=a$

Q10. $12x+5y\pm 26=0$ Q11. $2x-3y=6,\ -3x+2y=6$ Q12. $13x+13y=6$

Q13. $x-y+4=0,\ x+y-4=0$ Q15. $3x-y-5=0$

Q16. $3x+4y-18=0$ Q17. $2x+4y=11$ Q18. $x+2y\pm\sqrt{5}=0$

Q20. $3x-y=7,\ x+3y=9$ Q21. $(2+\sqrt{3})x-y-2\sqrt{3}-1=0,\ (2-\sqrt{3})x-y+2\sqrt{3}-1=0$

Q22. $52x+89y+519=0$ Q23. $\sqrt{\frac{2}{3}}$ units Q24. $y+1=x$

Q25. $\sqrt{3}x+y\pm 2=0$ Q26. $2x-9y+85=0$

Q27. (a) Slope-intercept form : $y=x+4$; Intercept form : $\frac{x}{-4}+\frac{y}{4}=1$

(b) Slope-intercept form : $y=\frac{1}{\sqrt{3}}x+\frac{8}{\sqrt{3}}$; Intercept form : $\frac{x}{-8}+\frac{y}{\frac{8}{\sqrt{3}}}=1$

Q28. $(\sqrt{3}+2)x+(2\sqrt{3}-1)y=8\sqrt{3}+1$ or $(\sqrt{3}-2)x+(2\sqrt{3}+1)y=8\sqrt{3}-1$

Q29. $x=-\frac{5}{22}$ Q30. $3x+y=20$ Q31. $x+\sqrt{3}y-\sqrt{3}=0$

Q32. $29x-2y=31$ Q33. $\left(\frac{13}{5},0\right)$ Q34. $x=1,\ y=1$

## Exercise 9.3

Q01. $\left(\frac{8}{\sqrt{3}},0\right),(0,8),4\text{ units},30^\circ,120^\circ$ Q03. $k^2$ Sq. units Q04. $\left(-\frac{1}{10},\frac{37}{10}\right)$

Q05. $(-16,-2)$ Q06. $(-1,-4)$ Q07. $\left(\frac{6}{5},\frac{7}{5}\right)$

Q11. Line is parallel to X-axis Q12. $30^\circ$ or $150^\circ$ Q13. (–2, 0), (8, 0)

Q14. $\left(0,-\frac{8}{3}\right),\left(0,\frac{32}{3}\right)$ Q15. $\frac{520}{17}$ units Q16. (a) $k=-2$ (b) $p=5$ Q17. $\frac{\pi}{6}$

Q18. $\frac{|\sin(\phi-\theta)|}{2\left|\sin\frac{(\phi-\theta)}{2}\right|}$ Q19. $\frac{1\pm 5\sqrt{2}}{7}$

## Exercise 9.4

Q01. $3x+y-11=0,\ 3y-x-3=0$ Q02. $3\sqrt{2}$ units Q03. $\frac{23}{18}\sqrt{5}$ units

Q04. $107x-3y=92$ Q06. $18x+12y+11=0$ Q07. $119x+102y=125$

Q08. $K=\frac{5}{9}(F-32)+273$, When $K=0$ then $F=-459.4$ Q09. $3x-4y+8=0$

Q10. $L=\frac{0.192}{90}(C-20)+124.942$ Q11. 1340 litres

Q13. $5y=3x+4,\ 7y=-x+16,\ 5y=3x-22,\ 7y=-x-10$ Q14. (1, 6)

**Exercise 9.5**

Q01. (a) $\cos\frac{3\pi}{4}x+\sin\frac{3\pi}{4}y=2\sqrt{2}$ (b) $x\cos\frac{2\pi}{3}+y\sin\frac{2\pi}{3}=4$ Q02. $\theta=\frac{7\pi}{6}$ and $p=1$

Q03. $x\cos 60^\circ+y\sin 60^\circ=5$, $x+\sqrt{3}y-10=0$ Q04. $\cos\frac{\pi}{4}x+\sin\frac{\pi}{4}y=\frac{1}{3\sqrt{2}}$

Q05. $x\cos 135^\circ+y\sin 135^\circ=6$, $x-y+6\sqrt{2}=0$

Q06. $x\cos 120^\circ+y\sin 120^\circ=10$, $y=\frac{1}{\sqrt{3}}x+\frac{20}{\sqrt{3}}$.

# CHAPTER 10

**Exercise 10.1**

Q01. $x^2+y^2-2ax-2by+2b^2=0$ Q02. $\left(\frac{1}{4},0\right)$, $r=\frac{1}{4}$ Q03. $x^2+y^2-7x+5y-14=0$

Q04. $x^2+y^2+6x+2y-90=0$ Q05. $x^2+y^2+x-2y-21=0$

Q06. $4x^2+4y^2-12x+16y-21=0$ Q07. $x^2+y^2+4x-21=0$, $x^2+y^2-12x+11=0$

Q08. $x^2+y^2-3x-2y-21=0$ Q09. $x^2+y^2-2x-4y=0$

Q10. $x^2+y^2+2x-3y=0$ Q11. $x^2+y^2-4x-6y-12=0$; $(2,3)$; 5 units

Q12. $x^2+y^2+4x+6y-12=0$ Q13. $x^2+y^2-4x-6y-87=0$

Q14. Inside the circle; since the distance of the point to the centre of the circle is less than the radius of the circle. Q18. $x^2+y^2-16x+4y-32=0$

Q19. $x^2+y^2-6x-8y+\frac{381}{169}=0$ Q20. $\frac{21\sqrt{3}}{4}$ Sq.units Q21. $x^2+y^2-4x-3y=0$

**Exercise 10.2**

Q01. (a) $(3,0)$; x-axis, $x=-3$; 12 (b) $(-2,0)$; x-axis; $x=2$; 8 (c) $\left(0,-\frac{9}{4}\right)$; y-axis; $y=\frac{9}{4}$; 9

Q02. (a) $y^2=24x$ (b) $y^2=-8x$ (c) $2x^2=25y$ Q03. $3x^2=-4y$

Q04. $y^2=12x$ Q05. $2y^2=9x$ Q06. $x^2+4xy+4y^2-22x+26y+16=0$

**Exercise 10.3**

Q01. (a) $(\pm\sqrt{7},0)$; $(\pm4,0)$; 8; 6; $\frac{\sqrt{7}}{4}$; $\frac{9}{2}$ (b) $(0,\pm\sqrt{5})$; $(0,\pm3)$; 6; 4; $\frac{\sqrt{5}}{3}$; $\frac{8}{3}$

Q02. (a) $\frac{x^2}{169}+\frac{y^2}{144}=1$ (b) $\frac{x^2}{9}+\frac{y^2}{4}=1$ (c) $\frac{x^2}{169}+\frac{y^2}{144}=1$ (d) $\frac{x^2}{10}+\frac{y^2}{40}=1$

Q03. $\frac{x^2}{144}+\frac{y^2}{128}=1$ Q04. $\frac{x^2}{52}+\frac{y^2}{13}=1$ Q05. $\frac{4x^2}{81}+\frac{4y^2}{45}=1$

Q06. $\frac{x^2}{64}+\frac{y^2}{100}=1$

**Exercise 10.4**

Q01. (a) $(\pm10,0)$; $(\pm6,0)$; $\frac{5}{3}$; $\frac{64}{3}$ (b) $\left(0\pm,\frac{\sqrt{17}}{4}\right)$; $(0,\pm1)$; $\frac{\sqrt{17}}{4}$; $\frac{1}{8}$

Q02. Transverse axis = 8; Conjugate axis = 2; $(0,\pm\sqrt{17})$; $(0,\pm4)$; $\frac{\sqrt{17}}{4}$; $\frac{1}{2}$

Q03. (a) $\frac{x^2}{4}-\frac{y^2}{5}=1$ (b) $3y^2-x^2=108$ (c) $y^2-x^2=5$ Q04. $9x^2-16y^2=144$

Q05. $\frac{y^2}{25}-\frac{x^2}{11}=1$ Q06. $7x^2-9y^2=343$ Q07. $45x^2-36y^2=80$

Q08. $x^2-y^2=32$ Q09. $\frac{16}{225}x^2+\frac{16}{625}y^2=1$

## Exercise 10.5

Q01. 4 cm from the vertex Q02. Focus is at the mid-point (5, 0) of the given diameter

Q03. $20\sqrt{3}$ cm Q04. $\sqrt{5}$ m Q05. $8a\sqrt{3}$ units Q06. 9.11 m (or $\frac{5694}{625}$ m)

Q07. 18 Sq.units Q09. $\frac{x^2}{81}+\frac{y^2}{9}=1$ Q10. $2\sqrt{6}$ m Q11. $\frac{x^2}{25}+\frac{y^2}{9}=1$

Q12. 1.56 m (Approx.) Q13. 60 cm Q14. (10, 0) Q15. $10\sqrt{7}$ m

Q16. 12 cm, 18 cm

# CHAPTER 11

## Exercise 11.1

Q01. OX′YZ, OXY′Z′, OXYZ′ Q02. 0 Q03. XZ-plane Q04. $\sqrt{549}$ units

Q06. $x=0,\ y=0$ Q07. 12 units

## Exercise 11.2

Q01. Yes, the triangle formed is an isosceles triangle

Q02. No, the triangle formed is not a right angled triangle Q07. 3

## Exercise 11.3

Q01. $(0,2,0)$ and $(0,-6,0)$ Q02. $x-2z=0$ Q03. $10x+6y-18z-29=0$

Q04. $9x^2+25y^2+25z^2-225=0$ Q05. $2x^2+2y^2+2z^2-4x-14y+4z+109=k^2$

Q06. (0, 5, 0) Q07. $\left(0,0,\frac{15}{11}\right)$ Q08. (3, 2, 0)

Q09. $\left(0,\frac{31}{16},-\frac{3}{16}\right)$ Q10. $1:2$ internally Q11. $\left(\frac{a}{2},\frac{b}{2},\frac{c}{2}\right)$

Q12. 5:3 Q13. 3:7 Q14. 3:13

## Exercise 11.4

Q01. $\left(-\frac{4}{5},\frac{1}{5},\frac{27}{5}\right)$, $(-8,17,3)$ Q02. $(6,-4,-2)$, $(8,-10,2)$ Q04. Points are collinear

Q05. $2:3$ externally, $(0,4,46)$ Q06. $2:3$ internally, $\left(0,\frac{19}{5},\frac{4}{5}\right)$ Q07. $(4,-2,6)$

Q08. $1:2$ internally Q09. $7,\sqrt{34},7$ Q10. $(1,-2,8)$ Q11. $3:5$ internally, $\left(-\frac{1}{2},\frac{11}{4},0\right)$

Q12. $5:7$ internally, $\left(\frac{29}{12},\frac{9}{4},\frac{25}{6}\right)$ Q13. $2:3$ internally Q14. $2:3$ internally, $\left(\frac{13}{5},-\frac{9}{5},-\frac{2}{5}\right)$

Q15. $3:4$ internally Q16. $\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3},\frac{z_1+z_2+z_3}{3}\right)$

Q17. (1, 1, 2) Q18. $(-2,5,8)$ Q19. (1, 1, –2) Q20. $a=-2, b=-\frac{16}{3}, c=2$

Q21. $a=-2, b=-8, c=2$ Q22. $\left(\frac{a}{2},\frac{b}{2},\frac{c}{2}\right)$ Q23. $(1,2,3),(3,4,5),(-1,6,-7)$

Q24. $\frac{\sqrt{1530}}{8}$ units Q25. $\left(-\frac{3}{10},\frac{25}{10},-\frac{2}{10}\right)$ Q26. $\left(\frac{38}{16},\frac{57}{16},\frac{17}{16}\right)$.

# CHAPTER 12

## Exercise 12.1

Q02. $e^k$

## Exercise 12.2

### Category I

Q01. (a) 5 (b) $4\pi$ (c) 1 (d) –4 (e) $-\frac{1}{4}$ (f) $\frac{2}{3\sqrt{3}}$

(g) $-\frac{1}{3}$ (h) $\frac{8}{5}$ (i) 2 (j) $\frac{1}{3}$ (k) $-\frac{4}{3}$

### Category II

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{5}{6a}(a+2)^{2/3}$ (d) $-\frac{1}{10}$ (e) $2\sqrt{a}$

(f) 2 (g) –2

### Category III

(a) $\frac{m^2}{n^2}$ (b) $\frac{1}{2}$ (c) $\frac{1}{8}$ (d) $\frac{1}{15}$ (e) $\frac{1}{4\sqrt{2}}$ (f) –4

(g) $-\frac{\sin\sqrt{a}}{4a^{3/2}}$ (h) $\frac{5}{2}$ (i) $\sqrt{2\operatorname{cosec}2}\cos 2$ (j) $-2\sqrt{a}\sin a$ (k) $\frac{1}{4\sqrt{2}}$

(l) 3 (m) 3 (n) $\frac{1}{3}$ (o) $\sec x(1+x\tan x)$ (p) $\frac{1}{4}$

(q) 4 (r) 1 (s) 2 (t) $\frac{3}{2}$ (u) 1 (v) $\frac{1}{2}$

(w) $2\cos 3$ (x) –12 (y) $\frac{1}{2}$ (z) $\frac{3}{2}$ (aa) $a^2\cos a+2a\sin a$

### Category IV

(a) $\frac{\sqrt{2}}{2}$ (b) 2 (c) $\frac{1}{\pi}$ (d) 2 (e) $\frac{1}{2}$ (f) $-2\sqrt{2}$

(g) $\sqrt{2}$ (h) $\frac{1}{36}$ (i) $-\frac{1}{8}$ (j) $\frac{\sqrt{3}}{3}$ (k) $\frac{1}{16}$ (l) –2

(m) 2 (n) $\frac{\sqrt{2}}{2}$ (o) $\frac{1}{4}$ (p) $\frac{1}{16}$ (q) $2\sqrt{3}$ (r) 4

(s) –3 (t) 0

### Category V

(a) $\frac{2}{3}$ (b) $\frac{2}{\pi}\log 3$ (c) $\frac{2}{\log 3}$ (d) $4\log 5$ (e) $\frac{3}{2}$ (f) $(\log a)^2$

(g) 1 (h) $\frac{2}{5}$ (i) $e^{-1}$ (j) $\frac{1}{2}$ (k) 1 (l) $\sqrt{2}$

(m) $\frac{\log 2}{\pi}$ (n) $\frac{2}{\pi}\log 2$ (o) $2e^2$ (p) $2\log 2$ (q) $\frac{3}{2}$

**Category VI**

(a) $\frac{1}{2}$ (b) $\frac{5}{2}$ (c) $\frac{1}{6}$ (d) 1 (e) $\frac{1}{2}$ (f) $\frac{1}{3}$ (g) $\frac{1}{3}$

**Category VII**

(a) $e^4$ (b) $\frac{1}{e}$ (c) $e^6$ (d) 1 (e) $e^2$ (f) $e^{-1}$

(g) $e^{-8}$ (h) $\frac{1}{e}$

## Exercise 12.3

Q01. $n = 5$ Q02. $k = \frac{8}{3}$ Q03. $m = \pm 1$ Q06. $a = 0, b = 4$

Q07. 6 Q08. 2 Q09. m and n can be any integer.

## Exercise 12.4

Q01. (a) $-\frac{13}{(x-5)^2}$ (b) $-\frac{16}{(7x-3)^2}$ (c) $1-\frac{1}{x^2}$ (d) $1+\frac{1}{x^2}$ (e) $\frac{1}{2\sqrt{x}}\left(1-\frac{1}{x}\right)$

(f) $-\frac{35}{2}(7x)^{-\frac{7}{2}}$ (g) $-\frac{15}{4}(-3x)^{\frac{1}{4}}$ (h) $-\frac{3}{x^4}$ (i) $2x+1$ (j) $-\frac{2}{(x-1)^2}$

(k) $\frac{1}{2}\cos\left(\frac{x}{2}\right)$ (l) $-\frac{1}{x^2}\cos\left(\frac{1}{x}\right)$ (m) $\sec^2 x$ (n) $\sin 2x$ (o) $-2x\sin x^2$

(p) $\frac{\cos x}{2\sqrt{\sin x}}$ (q) $\frac{\sec^2 x}{2\sqrt{\tan x}}$ (r) $\frac{\cos\sqrt{x}}{2\sqrt{x}}$ (s) $-\frac{\operatorname{cosec}^2\sqrt{x}}{2\sqrt{x}}$

(t) $3\sec 3x\tan 3x$ (u) $\cos(x+1)$ (v) $\cos x - \sin x$

(w) $x\sec^2 x + \tan x$ (x) $x\cos x + \sin x$ (y) $\frac{\cos x}{x} - \frac{\sin x}{x^2}$

(z) $\operatorname{cosec} x(1 - x\cot x)$ (aa) $\cos 2x$ (ab) $-\operatorname{cosec} x\cot x$

(ac) $-2\operatorname{cosec}\left(2x-\frac{\pi}{4}\right)\cot\left(2x-\frac{\pi}{4}\right)$ (ad) $-\sec(3-x)\tan(3-x)$

(ae) $-2\sin(2x+3)$ (af) $3\sec(3x+2)\tan(3x+2)$ (ag) $2x\cos x - x^2\sin x$

(ah) $-2x\sin(1+x^2)$ (ai) $-\cos(1-x)$ (aj) $2\tan x\sec^2 x$

Q02. (a) $4ax(ax^2+b)$ (b) $1-\frac{1}{x^2}$ (c) $b(n+1)(a+bx)^n$ (d) $\frac{x^3}{\sqrt{1+x^4}\sqrt{1+\sqrt{1+x^4}}}$

(e) $-5\sin x - 3\sin 2x + 14x\sec x^2\tan x^2 - \frac{15}{2x^4} + \frac{3}{2}\sqrt{x}$ (f) $\sec^2 x + \frac{1}{x^2}\operatorname{cosec}^2\left(\frac{1}{x}\right)$

(g) $\frac{ar - 2bpx - apx^2 - bq}{(px^2+qx+r)^2}$ (h) $n(ax+b)^n(a+bx)^n\left[\frac{a}{ax+b}+\frac{b}{a+bx}\right]$

(i) $-\frac{2}{(x+1)^2} - \frac{3x^2-2x}{(3x-1)^2}$ (j) $\frac{2(1-x^2)}{(x^2-x+1)^2}$ (k) $\frac{nx^{n-1}}{x-a} - \frac{x^n-a^n}{(x-a)^2}$

(l) $2x\sec^2(x^2+a^2)$ (m) $(x+\sec x)(1-\sec^2 x)+(x-\tan x)(1+\sec x\tan x)$

(n) $x^4(5\cos x + 3\sin x) + 4x^3(5\sin x - 3\cos x)$ (o) $-\frac{2}{(\sin x - \cos x)^2}$

(p) $\tan\frac{x}{2}\sec^2\frac{x}{2}$ (q) $\frac{21+(35x-9)\sin x+(9x+35)\cos x}{(3x+7\cos x)^2}$ (r) $\frac{1}{1+\tan x} - x\left(\frac{\sec x}{1+\tan x}\right)^2$

(s) $x(2-nx\cot x)\operatorname{cosec}^n x$ (t) $-\operatorname{cosec} x(2\operatorname{cosec}^2 2x+\cot x\cot 2x)$

(u) $\frac{1}{1+\cos x}$ (v) $-\frac{2\cos x}{(1+\sin x)^2}$ (w) $\cot x - x\operatorname{cosec}^2 x$

(x) $2x\cos x^2-2x^3\sin x^2+2x$ (y) $\tan x(1+\cos x)+\sec^2 x(x+\sin x)$

(z) $-\sec^2\left(\frac{\pi}{4}-x\right)$ (aa) $\frac{x^2}{(x\sin x+\cos x)^2}$ (ab) $-\cos a.\operatorname{cosec}^2 x$ (ac) $\frac{\pi x^2}{360}\sec^2 x^\circ + x\tan x^\circ$

### Exercise 12.5

Q01. –1 Q03. 1

# CHAPTER 13

### Exercise 13.1

Q01. 8.4 Q02. 6.32 Q03. 10.24 Q04. 4.56 Q05. 8.7
Q06. 5.1 Q07. 11.33

### Exercise 13.2

Q01. 66.2, 8.136

Q02. Mean, $\bar{x}=\frac{1}{n}(1+2+3+...+n)=\frac{n(n+1)}{2n}=\frac{(n+1)}{2}$

Variance, $\sigma^2=\frac{1}{N}\sum_{i=1}^{n} f_i x_i^2-(\bar{x})^2=\frac{1}{n}(1^2+2^2+3^2+...+n^2)-\left(\frac{n+1}{2}\right)^2=\frac{n^2-1}{12}$

Q03. S.D. = $\sqrt{43.4}=6.59$ Q04. 2276.1 Q05. Mean = 43.5, S.D. = 5.55.

### Exercise 13.3

Q02. 24, 12 Q03. 20 Q04. 4, 9 Q05. 4, 8 Q06. 6, 8
Q07. 3, 6 Q09. 6 Q10. 39.9, 5 Q11. (i) 10.1, 2.02 (ii) 10.2, 1.98
Q12. 20, 3.036 Q13. 6.5, 2.5 Q14. 40.045, 14.995

# CHAPTER 14

### Exercise 14.1

Q01. (i) {QH, QR, HQ, HR, RH, RQ} where Q denote a 1 rupee coin, H denotes a 2 rupee coin and R denotes a 5 rupee coin.
(ii) {0, 1, 2, …}

Q02. {$HB_1$, $HB_2$, $HB_3$, $HW_1$, $HW_2$, $HW_3$, $HW_4$, T1, T2, T3, T4, T5, T6}

Q03. {H, TH, TTH, TTTH, …} Q04. (i) {BB, BG, GB, GG} (ii) {0, 1, 2}

Q05. {RW, WR, WW} Q06. {HH, HT, T1, T2, T3, T4, T5, T6}

Q07. {DDD, DDN, DND, NDD, DNN, NDN, NND, NNN}

Q08. {T, H1, H3, H5, H21, H22, H23, H24, H25, H26, H41, H42, H43, H44, H45, H46, H61, H62, H63, H64, H65, H66}

Q09. {(1,2), (1,3), (1,4), (2,1), (2,3), (2,4), (3,1), (3,2), (3,4), (4,1), (4,2), (4,3)}

Q10. {1HH,1HT,1TH,1TT,2H,2T,3HH,3HT,3TH,3TT,4H,4T,5HH,5HT,5TH,5TT,6H,6T}

Q11. {$TR_1$, $TR_2$, $TB_1$, $TB_2$, $TB_3$, H1, H2, H3, H4, H5, H6}

Q12. {6, (1,6), (2,6), (3,6), (4,6), (5,6), (1,1,6), (1,2,6), ..., (1,5,6), (2,1,6). (2,2,6), ..., (2,5,6), ..., (5,1,6), (5,2,6), ... }

Q13. (a) Car (Ferry, Train, Helicopter, Air Craft), Bus (Ferry, Train, Helicopter, Air Craft), Train (Ferry, Train, Helicopter, Air Craft) (b) 12 ways.

### Exercise 14.2

Q01. C and D Q02. No

Q03. (i) {1, 2, 3, 4, 5, 6} (ii) $\phi$ (iii) {3, 6} (iv) {1, 2, 3} (v) {6}
(vi) {3, 4, 5, 6}, A∪B = {1, 2, 3, 4, 5, 6}, A∩B = $\phi$, B∪C = {3, 6}, E∩F = {6}, D∩E = $\phi$, A – C = {1, 2,4,5}, D – E = {1,2,3}, F′ = {1, 2}, E∩F′ = $\phi$.

Q04. (i) A and B; A and C; B and C; C and D (ii) A and C (iii) B and D

Q05. A = {(2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)},
B = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6)}
C = {(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,2), (4,1)}
(i) A′ = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6)} = B
(ii) B′ = {(2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)} = A
(iii) A∪B = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (2,1), (2,2), (2,3), (2,5), (2,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)} = S
(iv) A ∩ B = ϕ
(v) A – C = {(2,4), (2,5), (2,6), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}
(vi) B ∪ C = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6)}
(vii) B ∩ C = {(1,1), (1,2), (1,3), (1,4), (3,1), (3,2)}
(viii) A∩B′∩C′ = {(2,4),(2,5),(2,6),(4,2),(4,3),(4,4),(4,5),(4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

Q06. (i) $(E\cap F\cap G)\cup(E'\cap F\cap G)\cup(E\cap F'\cap G)\cup(E\cap F\cap G')$
(ii) $(E\cap F'\cap G')\cup(E'\cap F\cap G')\cup(E'\cap F'\cap G)$ (iii) $(E\cap F'\cap G')$
(iv) $(E'\cap F\cap G)\cup(E\cap F'\cap G)\cup(E\cap F\cap G')$

Q07. Impossible event Q08. Mutually exclusive events Q09. 0 Q10. 1

## Exercise 14.3

Q01. (i) $\frac{4}{9}$ (ii) $\frac{2}{9}$ (iii) $\frac{3}{9}$ (iv) $\frac{6}{9}$ (v) $\frac{7}{9}$ Q02. (a) 0.87 (b) 0.98 (c) 0.11

Q03. (a) $\frac{1}{6}$ (b) $\frac{2}{3}$ (c) $\frac{1}{6}$ Q04. (i) $\frac{1}{4}$ (ii) $\frac{12}{13}$ (iii) $\frac{1}{2}$ (iv) $\frac{3}{4}$ (v) $\frac{1}{2}$

Q05. (i) $\frac{1}{12}$ (ii) $\frac{1}{12}$ Q06. $\frac{3}{5}$

Q07. ₹4.00 gain, ₹1.50 gain, ₹1.00 loss, ₹3.50 loss, ₹6.00 loss. Therefore P(Winning ₹4.00) $=\frac{1}{16}$,
P(Winning ₹1.50) $=\frac{1}{4}$, P(Losing ₹1.00) $=\frac{3}{8}$, P(Losing ₹3.50) $=\frac{1}{4}$, P(Losing ₹6.00) $=\frac{1}{16}$.

Q08. (i) $\frac{6}{13}$ (ii) $\frac{7}{13}$ Q09. $\frac{1}{38760}$ Q10. 0.6 or 60% Q11. 0.55

Q12. 0.65 Q13. (i) $\frac{19}{30}$ (ii) $\frac{11}{30}$ (iii) $\frac{4}{30}$

Q14. (i) $\frac{12}{24}$ (ii) $\frac{4}{24}$ (iii) $\frac{2}{24}$ (iv) $\frac{12}{24}$
Can we expect you to find the probability in case of (v)?

Q15. (i) $\frac{{}^4C_4\times{}^{48}C_3}{{}^{52}C_7}=\frac{1}{7735}$ (ii) $\frac{{}^4C_3\times{}^{48}C_4}{{}^{52}C_7}=\frac{9}{1547}$ (iii) $\frac{9}{1547}+\frac{1}{7735}=\frac{46}{7735}$.

Q17. (a) $\frac{1}{60}$ (b) $\frac{6}{60}$ Q18. (i) $\frac{1}{2}$ (ii) $\frac{1}{2}$ (iii) $\frac{5}{6}$

Q19. (a) $\frac{999}{1000}$ (b) $\frac{^{9990}C_2}{^{10000}C_2}$ (c) $\frac{^{9990}C_{10}}{^{10000}C_{10}}$ Q20. (a) $\frac{17}{33}$ (b) $\frac{16}{33}$ Q21. $\frac{2}{3}$

Q22. (i) $\frac{99}{249}$ (ii) $\frac{3}{8}$ Q23. $\frac{1}{5040}$ Q24. $\frac{260}{447}$

Q25. (i) $\frac{6}{36}$ (ii) $\frac{18}{36}$ (iii) $\frac{15}{36}$ (iv) $\frac{11}{36}$ Q26. $\frac{1}{3}, \frac{1}{15}, \frac{1}{30}$ Q27. $\frac{28}{36}$

Q28. $\frac{233}{500}$ Q30. $\frac{13}{30}$ Q31. 3% Q32. (i) $\frac{4}{56}$ (ii) $\frac{15}{56}$

Q33. $\frac{23}{28}$ Q34. $\frac{6}{10}, \frac{3}{10}, \frac{1}{10}$.

# MULTIPLE CHOICE QUESTIONS

## ☑ Chapter 01

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 01. | (c) | 02. | (c) | 03. | (a) | 04. | (a) | 05. | (c) | 06. | (d) | 07. | (c) |
| 08. | (a) | 09. | (b) | 10. | (c) | 11. | (b) | 12. | (b) | 13. | (d) | 14. | (c) |
| 15. | (b) | 16. | (a) | 17. | (c) | 18. | (d) | 19. | (c) | 20. | (b) | 21. | (a) |
| 22. | (c) | 23. | (d) | 24. | (c) | 25. | (d) | 26. | (a) | 27. | (b) | 28. | (c) |
| 29. | (b) | 30. | (d) | 31. | (b) | 32. | (c) | 33. | (a) | 34. | (d) | 35. | (b) |
| 36. | (d) | 37. | (b) | | | | | | | | | | |

## ☑ Chapter 02

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 01. | (b) | 02. | (d) | 03. | (b) | 04. | (d) | 05. | (c) | 06. | (a) | 07. | (b) |
| 08. | (c) | 09. | (a) | 10. | (d) | 11. | (a) | 12. | (d) | 13. | (c) | 14. | (a) |
| 15. | (c) | 16. | (c) | 17. | (b) | 18. | (a) | 19. | (d) | 20. | (d) | 21. | (c) |
| 22. | (d) | 23. | (d) | 24. | (c) | 25. | (d) | 26. | (d) | 27. | (c) | 28. | (d) |
| 29. | (b) | 30. | (c) | 31. | (c) | 32. | (d) | 33. | (d) | 34. | (b) | 35. | (c) |
| 36. | (d) | 37. | (c) | 38. | (b) | 39. | (c) | 40. | (c) | | | | |

## ☑ Chapter 03

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 01. | (d) | 02. | (a) | 03. | (c) | 04. | (c) | 05. | (b) | 06. | (c) | 07. | (c) |
| 08. | (a) | 09. | (c) | 10. | (c) | 11. | (c) | 12. | (a) | 13. | (d) | 14. | (c) |
| 15. | (c) | 16. | (c) | 17. | (a) | 18. | (d) | 19. | (d) | 20. | (c) | 21. | (c) |
| 22. | (a) | 23. | (b) | 24. | (b) | 25. | (c) | 26. | (d) | 27. | (d) | 28. | (c) |
| 29. | (b) | 30. | (a) | 31. | (b) | 32. | (d) | 33. | (d) | 34. | (d) | 35. | (c) |
| 36. | (b) | 37. | (c) | 38 | (b) | 39. | (b) | 40. | (b) | 41. | (c) | 42. | (d) |
| 43. | (c) | 44. | (a) | 45. | (c) | 46. | (b) | | | | | | |

## ☑ Chapter 04

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 01. | (c) | 02. | (a) | 03. | (b) | 04. | (a) | 05. | (c) | 06. | (a) | 07. | (c) |
| 08. | (a) | 09. | (a) | 10. | (d) | 11. | (c) | 12. | (b) | 13. | (b) | 14. | (b) |
| 15. | (c) | 16. | (a) | 17. | (b) | 18. | (a) | 19. | (d) | 20. | (b) | 21. | (a) |
| 22. | (a) | 23. | (b) | 24. | (c) | 25. | (d) | 26. | (d) | 27. | (b) | 28. | (c) |
| 29. | (b) | 30. | (a) | 31. | (c) | 32. | (a) | 33. | (c) | | | | |

## ☑ Chapter 05

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 01. | (b) | 02. | (b) | 03. | (c) | 04. | (c) | 05. | (b) | 06. | (c) | 07. | (c) |
| 08. | (d) | 09. | (c) | 10. | (d) | 11. | (d) | 12. | (c) | 13. | (c) | 14. | (c) |
| 15. | (a) | 16. | (d) | 17. | (c) | 18. | (b) | 19. | (c) | 20. | (b) | 21. | (a) |
| 22. | (c) | 23. | (d) | 24. | (a) | | | | | | | | |

## ☑ Chapter 06

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 01. | (a) | 02. | (c) | 03. | (c) | 04. | (b) | 05. | (d) | 06. | (b) | 07. | (c) |
| 08. | (a) | 09. | (a) | 10. | (b) | 11. | (d) | 12. | (c) | 13. | (d) | 14. | (a) |
| 15. | (b) | 16. | (c) | 17. | (c) | 18. | (d) | 19. | (c) | 20. | (d) | 21. | (a) |
| 22. | (c) | 23. | (b) | 24. | (b) | 25. | (c) | 26. | (b) | 27. | (c) | | |

## ☑ Chapter 07

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 01. | (b) | 02. | (b) | 03. | (a) | 04. | (d) | 05. | (c) | 06. | (c) | 07. | (b) |
| 08. | (b) | 09. | (d) | 10. | (c) | 11. | (c) | 12. | (c) | 13. | (b) | 14. | (b) |
| 15. | (c) | 16. | (b) | 17. | (d) | 18. | (b) | 19. | (a) | 20. | (d) | 21. | (c) |
| 22. | (b) | 23. | (a) | | | | | | | | | | |

## ☑ Chapter 08

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 01. | (c) | 02. | (c) | 03. | (b) | 04. | (b) | 05. | (a) | 06. | (c) | 07. | (b) |
| 08. | (d) | 09. | (c) | 10. | (a) | 11. | (c) | 12. | (c) | 13. | (a) | 14. | (b) |
| 15. | (a) | 16. | (b) | 17. | (c) | 18. | (d) | 19. | (a) | 20. | (a) | 21. | (d) |
| 22. | (c) | 23. | (b) | 24. | (b) | 25. | (c) | | | | | | |

## ☑ Chapter 09

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 01. | (a) | 02. | (a) | 03. | (c) | 04. | (b) | 05. | (b) | 06. | (d) | 07. | (d) |
| 08. | (c) | 09. | (a) | 10. | (b) | 11. | (a) | 12. | (a) | 13. | (d) | 14. | (a) |
| 15. | (a) | 16. | (b) | 17. | (b) | 18. | (c) | 19. | (d) | 20. | (a) | 21. | (b) |
| 22. | (b) | 23. | (a) | 24. | (c) | 25. | (a) | 26. | (b) | 27. | (d) | 28. | (d) |
| 29. | (b) | 30. | (a) | 31. | (c) | | | | | | | | |

## ☑ Chapter 10

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 01. | (c) | 02. | (c) | 03. | (b) | 04. | (a) | 05. | (d) | 06. | (b) | 07. | (d) |
| 08. | (b) | 09. | (b) | 10. | (a) | 11. | (b) | 12. | (c) | 13. | (a) | 14. | (c) |

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 15. | (d) | 16. | (c) | 17. | (a) | 18. | (c) | 19. | (b) | 20. | (d) | 21. | (b) |
| 22. | (a) | 23. | (d) | 24. | (b) | 25. | (d) | | | | | | |

### ☑ Chapter 11

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 01. | (c) | 02. | (d) | 03. | (a) | 04. | (c) | 05. | (a) | 06. | (b) | 07. | (a) |
| 08. | (b) | 09. | (d) | 10. | (d) | 11. | (d) | 12. | (c) | 13. | (d) | 14. | (b) |
| 15. | (d) | 16. | (a) | 17. | (c) | 18. | (a) | 19. | (a) | 20. | (c) | | |

### ☑ Chapter 12

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 01. | (c) | 02. | (d) | 03. | (c) | 04. | (b) | 05. | (c) | 06. | (c) | 07. | (b) |
| 08. | (d) | 09. | (c) | 10. | (c) | 11. | (b) | 12. | (a) | 13. | (b) | 14. | (c) |
| 15. | (b) | 16. | (a) | 17. | (d) | 18. | (c) | 19. | (d) | 20. | (c) | 21. | (b) |
| 22. | (a) | 23. | (a) | 24. | (b) | 25. | (a) | 26. | (c) | 27. | (d) | 28. | (a) |
| 29. | (a) | 30. | (c) | 31. | (b) | 32. | (c) | 33. | (d) | 34. | (b) | 35. | (b) |

### ☑ Chapter 13

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 01. | (d) | 02. | (c) | 03. | (d) | 04. | (c) | 05. | (b) | 06. | (c) | 07. | (b) |
| 08. | (d) | 09. | (b) | 10. | (b) | 11. | (b) | 12. | (a) | 13. | (b) | 14. | (a) |
| 15. | (c) | 16. | (c) | 17. | (c) | 18. | (a) | 19. | (b) | 20. | (d) | 21. | (b) |
| 22. | (c) | | | | | | | | | | | | |

### ☑ Chapter 14

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 01. | (c) | 02. | (d) | 03. | (b) | 04. | (c) | 05. | (a) | 06. | (d) | 07. | (d) |
| 08. | (c) | 09. | (d) | 10. | (d) | 11. | (d) | 12. | (b) | 13. | (c) | 14. | (c) |
| 15. | (b) | 16. | (c) | 17. | (d) | 18. | (d) | 19. | (c) | 20. | (d) | 21. | (b) |
| 22. | (a) | 23. | (c) | 24. | (a) | | | | | | | | |

## ASSERTION REASON BASED QUESTIONS

### ☑ Unit-I

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 01. | (a) | 02. | (b) | 03. | (d) | 04. | (a) | 05. | (b) | 06. | (c) | 07. | (d) |
| 08. | (d) | 09. | (a) | 10. | (a) | 11. | (d) | 12. | (a) | 13. | (c) | 14. | (a) |
| 15. | (d) | 16. | (a) | 17. | (d) | 18. | (a) | 19. | (c) | 20. | (a) | 21. | (d) |
| 22. | (d) | 23. | (c) | 24. | (d) | 25. | (a) | 26. | (b) | 27. | (b) | 28. | (a) |
| 29. | (a) | 30. | (d) | 31. | (a) | 32. | (b) | 33. | (a) | 34. | (d) | 35. | (a) |
| 36. | (a) | 37. | (a) | 38. | (a) | 39. | (a) | 40. | (d) | | | | |

### ☑ Unit-II

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 01. | (a) | 02. | (d) | 03. | (a) | 04. | (b) | 05. | (a) | 06. | (c) | 07. | (b) |
| 08. | (a) | 09. | (c) | 10. | (a) | 11. | (a) | 12. | (a) | 13. | (d) | 14. | (d) |
| 15. | (b) | 16. | (d) | 17. | (b) | 18. | (b) | 19. | (a) | 20. | (c) | | |

### ☑ Unit-III

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 01. | (a) | 02. | (c) | 03. | (a) | 04. | (b) | 05. | (c) | 06. | (d) | 07. | (b) |
| 08. | (a) | 09. | (b) | 10. | (b) | 11. | (d) | 12. | (d) | 13. | (a) | 14. | (d) |
| 15. | (b) | 16. | (c) | 17. | (d) | 18. | (a) | 19. | (c) | 20. | (a) | 21. | (c) |
| 22. | (b) | 23. | (a) | 24. | (b) | 25. | (b) | 26. | (a) | 27. | (b) | 28. | (b) |
| 29. | (c) | 30. | (c) | | | | | | | | | | |

### ☑ Unit-IV

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 01. | (a) | 02. | (d) | 03. | (d) | 04. | (c) | 05. | (a) | 06. | (a) | 07. | (a) |
| 08. | (b) | 09. | (b) | 10. | (b) | 11. | (c) | | | | | | |

### ☑ Unit-V

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 01. | (a) | 02. | (c) | 03. | (a) | 04. | (b) | 05. | (c) | 06. | (d) | 07. | (c) |
| 08. | (a) | 09. | (a) | 10. | (c) | 11. | (a) | 12. | (a) | 13. | (c) | | |

## CASE STUDY & PASSAGE BASED QUESTIONS

### ☑ Chapter 01

| | | | | | |
|---|---|---|---|---|---|
| Q01. | (i) 35 | (ii) 11 | (iii) 11 | (iv) 18 | (v) 10 |
| Q02. | (i) 3300 | (ii) 600 | (iii) 5200 | (iv) 200 | (v) 6000 |
| Q03. | (i) 275 | (ii) 325 | (iii) 50 | (iv) 125 | (v) 375 |
| Q04. | (i) 6 | (ii) 3 | (iii) 9 | (iv) 6 | (v) 3 |
| Q05. | (i) 6 | (ii) 1 | (iii) 16 | (iv) 32 | (v) 8 |
| Q06. | (i) 450 | (ii) 20 | (iii) 190 | (iv) 95 | (v) 50 |
| Q07. | (i) 20 | (ii) 47 | (iii) 45 | (iv) 53 | (v) 10 |
| Q08. | (i) 30 | (ii) 52 | (iii) 8 | (iv) 19 | (v) 10 |
| Q09. | (i) d | (ii) {A, C, E, H, I, M, O, S, T} | (iii) {A, C, E, H, I, J, L, M, N, O, S, T, V, Y} | | |

(iv) $2^{11}-1$ (v) $2^{12}$

Q10. (i) 3300 (ii) 4000 (iii) 500 (iv) 5200 (v) 6000

Q11. (i) 8 (ii) Yes (iii) $\{-2,-1\},\{-1, 0\},\{-2, 0\}$ (iv) $\{-2\},\{-1\},\{0\}$
(v) $\{\ \}, \{-2\}, \{-1\}, \{0\}, \{-2,-1\}, \{-1, 0\}, \{-2, 0\}, \{-2,-1, 0\}$

Q12. (i) $\{W,I,S,H\}$ (ii) $\{D,S,H\}$ (iii) $\{A,D,H,I,N,S,T,V\}$ (iv) $\{I,S,H\}$

Q13. (i) 57 (ii) 27 (iii) 12 (iv) 23

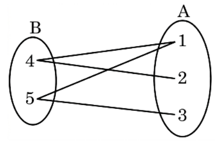
Q14. (i) 80 (ii) 35 (iii) 20 (iv) 20

Q15. (i) 20 (ii) 30 (iii) 20 (iv) 20.

## ☑ Chapter 02

Q01. (i) It is not a function as for element 4 in the domain, there is no image in the codomain.
(ii) It is not a function as for element 2 in the domain, there are two images 1 and 2 in the codomain.
(iii) Domain : $[-2, 8)$ ; Range : $[-1, 3]\cup\{4\}$.
(iv) Integral values of x for which $f(x)=3$ are $x=\{-1, 0, 1, 2\}$.
(v) Since $(b, x),(b, y)\in f$. Therefore, f is not a function as b has two images x and y.
Recall that, for f to be a function it should have unique image in the codomain.

Q02. (i) $R = A\times B$. (ii) $2^6$.
(iii) Note that $(4,5)\in S$ but $4\notin A$. So, S is not a relation from A to B.
(iv) $R'=\{(4,1),(4,2),(5,1)\}$.
(v) Arrow diagram for $R'$ is given below.




Q03. (i) $\{(1,0),(1,1),(1,2),(2,0),(2,1),(2,2),(3,0),(3,1),(3,2),(4,0),(4,1),(4,2)\}$
(ii) $\{(0,1), (0,2), (1,1),(1,2),(2,1),(2,2),(3,1),(3,2),(4,1),(4,2)\}$
(iii) $R=\{(2,2),(3,1),(4,0)\}$ (iv) $\{2,3,4\}$ (v) $\{0, 1, 2\}$

Q04. (i) $g(x)=\begin{cases}-2x, \text{ if } x<-1\\ 2, \text{ if } -1\le x<1\\ 2x, \text{ if } x\ge 1\end{cases}$ (ii) R (iii) a (iv) a (v) $[2, \infty)$

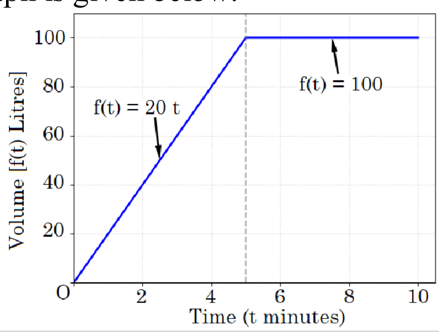
Q05. (i) $15000+30x$ (ii) $45x$ (iii) $15(x-1000)$ (iv) 1000 (v) 60000

Q06. (i) $y=x$ (ii) 50 (iii) 40 min (iv) $y=50$ (v) 40 min

Q07. (i) $[1, \infty)$ (ii) $R-\left\{\frac{3}{2}\right\}$ (iii) $[1, \infty)$ (iv) $(1, \infty)$ (v) $[1, \infty)-\left\{\frac{3}{2}\right\}$

Q08. (i) $x\in[0, 8]$ (ii) $[10, 50]$ (iii) 25 minutes
(iv) Distance $=5$ km. Since $x=5\in[0, 8]$ so, the answer is valid in the given context.

Q09. (i) Domain $= t\in[0, 10]$; range $=[0, 100]$
(ii) Graph is given below.



(iii) 60 L and 100 L (iv) $5 \le t \le 10$ minutes.

Q10. (i) [0, 5] (ii) [5, 20] (iii) 20 Mbps, 10 Mbps (iv) 4 GB

## ☑ Chapter 03

Q01. (i) $\frac{120}{169}$ (ii) $\frac{119}{169}$ (iii) $\frac{5}{\sqrt{26}}$ (iv) $-\frac{1}{\sqrt{26}}$ (v) $-5$

Q02. (i) $-\frac{n}{m}$ (ii) $-\frac{2mn}{m^2+n^2}$ (iii) $\frac{m^2-n^2}{m^2+n^2}$ (iv) $\frac{2mn}{n^2-m^2}$ (v) $\frac{2-m^2-n^2}{2}$

Q03. (i) $\frac{2\pi}{5}$ (ii) 70 m (iii) 176 m (iv) 132 m (v) $2:3$

Q04. (i) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (ii) $\frac{10\sqrt{3}+10}{\sqrt{3}-1}$ (iii) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (iv) $10\sqrt{2}(\sqrt{3}+1)$ (v) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

Q05. (i) $R-(2n\pm 1)\pi,\ n\in Z$ (ii) $R-(4n-1)\frac{\pi}{4},\ n\in Z$ (iii) [1, 3]

(iv) {–1, 0, 1} (v) [2, 9]

Q06. (i) $\tan 56°$ (ii) $\cot 54°$ (iii) $\frac{1}{\sqrt{3}}$ (iv) $\sqrt{3}$ (v) $\tan 82°$

Q07. (i) $-\frac{56}{65}$ (ii) $\frac{33}{65}$ (iii) $-\frac{56}{33}$ (iv) $\frac{63}{65}$ (v) $-\frac{16}{65}$

Q08. (i) $f(x)=\sin x$ (ii) $f(x)=\cos x$ (iii) Range $=[-1, 1]$ (iv) 0 .

## ☑ Chapter 04

Q01. (i) $-\frac{1}{2}+\frac{i}{2}$ (ii) $x=-\frac{1}{4},\ y=\frac{7}{2}$ (iii) $\frac{1-2i}{1+3i}$ (iv) $\frac{1}{\sqrt{2}}$ (v) II Quadrant

Q02. (i) no value of $x$ (ii) $z=0$ (iii) $z=\bar{z}$ (iv) $z=-\bar{z}$ (v) $\sqrt{3}+i\sqrt{2}$

Q03. (i) 5 units (ii) $\tan^{-1}\frac{4}{3}$ (iii) $3-4i$ (iv) $\frac{3}{25}-i\frac{4}{25}$

Q04. (i) 2 units (ii) $-\frac{\pi}{3}$ (iii) $1+\sqrt{3}i$ (iv) $\frac{1}{4}+i\frac{\sqrt{3}}{4}$.

## ☑ Chapter 05

Q01. (i) d (ii) a (iii) c (iv) b (v) c

Q02. (i) $26000+30x$ (ii) $43x$ (iii) $-26000+13x$ (iv) more than 2000 (v) 2600

Q03. (i) c (ii) d (iii) c (iv) a (v) c

Q04. (i) a (ii) c (iii) b (iv) d (v) a

## ☑ Chapter 06

Q01. (i) ${}^{22}C_7$ (ii) ${}^{22}C_{10}$ (iii) ${}^{22}C_7+{}^{22}C_{10}$ (iv) 720 (v) 60

Q02. (i) $9^6$ (ii) $6\times 10^6$ (iii) ${}^{9}P_6$ (iv) $10^6$ (v) $6\times 10^5$

Q03. (i) 360 (ii) 120 (iii) 240 (iv) 20 (v) 120

Q04. (i) ${}^{14}C_3\times{}^{13}C_2$ (ii) 6930 (iii) $14!\times 13!\times 2$ (iv) $12\times{}^{14}C_3$ (v) 18876

Q05. (i) ${}^{8}P_8$ or $8!$ (ii) $7!$ (iii) $3!\times 6!$ (iv) $6!$ (v) 1607

Q06. (i) 420 (ii) 285

Q07. (i) 8 (ii) 21 (iii) 10 (iv) 1680

Q08. (i) 10 (ii) 9 (iii) 28 (iv) 2520 .

## ☑ Chapter 07

Q01. (i) $a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$

(ii) $\sum_{r=0}^{6} {}^6C_r a^{6-r} b^r$ (iii) $a^6$ (iv) fourth term (v) $20a^3b^3$

Q02. (i) ${}^6C_r x^{6-r} y^r$ (ii) 15 (iii) 15 (iv) 7.

## ☑ Chapter 08

Q01. (i) Geometric progression (ii) 30 (iii) $(30)(2)^n$ (iv) $(30)(2)^4$ (v) $(30)(2)^6$

Q02. (i) $3\times 4^{\frac{1}{n+1}}$ (ii) 6 (iii) 18 (iv) $x=6$, $y=24$

Q03. (i) 750 lumens, 562.5 lumens, 421.875 lumens (ii) 0.75
(iii) 273.3 lumens (iv) $4000(1-0.75^5)$ lumens

Q04. (i) $r=2$ (ii) ₹160000 (iii) 50000 (iv) 40000.

## ☑ Chapter 09

Q01. (i) (5, 3) (ii) $5x-3y+23=0$ (iii) $5x-3y-16=0$
(iv) $3x+5y-30=0$ (v) 39 sq.units

Q02. (i) $\frac{2}{3}$ (ii) $\frac{5}{3}$ (iii) $\left(-\frac{1}{17}, \frac{22}{17}\right)$ (iv) a (v) $119x+102y=125$

Q03. (i) (0, 0) (ii) (k, –k) (iii) (k, k) (iv) $k^2$ Sq. units (v) AB and AC

Q04. (i) $x+y=2$ (ii) $\frac{3}{2}\sqrt{2}$ units (iii) $\left(\frac{1}{2}, \frac{3}{2}\right)$ (iv) 3 sq.units (v) $\left(\frac{10}{3}, \frac{10}{3}\right)$

Q05. (i) $\frac{1}{2}$ (ii) $x-2y+4=0$ (iii) (4, 4) (iv) $\sqrt{5}$ units (v) (6, 5)

Q06. (i) 4 (ii) 4 (iii) $y=4x-7992$ (iv) Rs.104

## ☑ Chapter 10

Q01. (i) $2\sqrt{6}$ m (ii) $y=\frac{x^2}{1200}$ (iii) $\frac{8}{300}$ (iv) 1200 (v) $\frac{1}{400}$

Q02. (i) Parabola opening upwards (ii) 6 units (iii) 72 sq.units
(iv) 24 units (v) 24 units

## ☑ Chapter 11

Q01. (i) $\sqrt{2}$ (ii) $\sqrt{3}$ (iii) $\Delta ABC$ is right angled triangle
(iv) square (v) (0, 2, 0) and (0, –6, 0)

Q02. (i) $|c|$ (ii) $\sqrt{b^2+c^2}$ (iii) (0, b, c) (iv) (0, 0, c) (v) $2|b|$

## ☑ Chapter 12

Q01. (i) $x+y=25$ (ii) $25x-x^2$ (iii) $25-2x$ (iv) 12.5 (v) 0

Q02. (i) $\frac{1}{2\sqrt{x}}$ (ii) $-\text{cosec}^2 x$ (iii) $\frac{-2x\,\text{cosec}^2 x+\cot x}{2\sqrt{x}}$ (iv) $\frac{\cot x+2x\,\text{cosec}^2 x}{2\sqrt{x}\cot^2 x}$
(v) $\frac{-(2x\,\text{cosec}^2 x+\cot x)}{2x^{3/2}}$

Q03. (i) 5 m/s (ii) $\lim_{t\to 2} v(t)$ is defined (iii) 1 $m/s^2$.

## ☑ Chapter 13

Q01. (i) 12181.82 (ii) 12000 (iii) $6^{th}$ (iv) 1090.91 (v) 5000

Q02. (i) 51 (ii) 105 (iii) 5.92 (iv) 2
(v) He should invest in the share Y

☑ **Chapter 14**

Q01. (i) 0.87 (ii) 0.11

Q02. (i) $\frac{^{13}C_3 \times {^{13}C_1}}{^{52}C_4}$ (ii) $\frac{^{4}C_2}{^{52}C_2}$

Q03. (i) 7 (ii) $\frac{3}{7}$ (iii) $\frac{1}{7}$ (iv) $\frac{3}{7}$ (v) Yoga and Rope Skipping

Q04. (i) $\frac{19}{30}$ (ii) 22 (iii) $\frac{11}{30}$ (iv) $\frac{2}{15}$ (v) $\frac{1}{10}$.

Q05. (i) $\frac{4}{5}$ (ii) $\frac{7}{20}$ (iii) $\frac{1}{5}$ (iv) $\frac{1}{5}$.

# TABLE OF CONTENTS

# Chapter 01 SOLUTIONS

## Exercise 1.1

Q01. (a) There are no fixed criteria mentioned in this statement for calling a boy fat. That is, it is not well defined statement, so it is not a set.

(b) There are no fixed criteria mentioned in this statement for calling a girl beautiful. That is, it is not well defined, so it is not a set.

(c) The collection of Maths teachers in your school is defined. So, it is a set.

(d) It is not well defined, so it is not a set, (as level of difficulty depends upon the individual).

(e) It is not well defined, so it is not a set, (as there is no fixed def. of smartness for boys).

Q02. (a) If x is non-negative integer then, $x = 0, 1, 2, \ldots$.

Also, $x^2 < 50$ so, maximum value of x can be 7.

Therefore, the tabular form is $\{0,1,2,3,4,5,6,7\}$.

(b) Since $x^2 + x - 2 = 0$ implies $(x+2)(x-1) = 0$

$\therefore x = -2, 1 \in Z$

Therefore, the roster form is $\{-2, 1\}$.

(c) Letters in the word COMBINATIONS are C, O, M, B, I, N, A, T, S.

Therefore, the roster form is {C, O, M, B, I, N, A, T, S}.

(d) Since x is an odd integer and $3 \le x < 13$ so, value of x can be 3, 5, 7, 9 and 11.

Therefore, the roster form is $\{3,5,7,9,11\}$.

(e) $4x + 9 < 39$ means $x < \frac{30}{4}$; also $x \in N$. Then value of x can be 1, 2, 3, …, 7.

Therefore, the roster form is $\{1,2,3,4,5,6,7\}$.

(f) As $x \in Z^+$ so x can be 1, 2, 3, …

Moreover $|x - 3| < 7$ implies, $x = -3, -2, -1, 0, 1, 2, 3, \ldots, 9$.

Clearly, value of x for the required set can be 1, 2, 3, …, 9.

Therefore, the roster form is $\{1, 2, \ldots, 9\}$.

(g) Two digit numbers start from 10 and end at 99.

The first such no., the sum of whose digits is nine is 18, then 27 and so on till 90.

The required tabular form is {18, 27, 36, 45, 54, 63, 72, 81, 90}.

(h) The vowels in English alphabet are $a, e, i, o, u$. The vowels which precede r are $a, e, i, o$.

Therefore, the tabular form is $\{a, e, i, o\}$.

(i) Note that, here $n = 1, 2, 3, 4, 5$ $\quad \therefore x = \frac{1}{1}, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}$.

Therefore, the roster form is $\left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}\right\}$.

(j) {x : x is a prime number which is divisor of 60} = {2, 3, 5} $\quad (\because 60 = 2^2 \times 3 \times 5$

Q03. **Note** that, while writing the set-builder form for a given tabular form of a set, we may get different answers. But in all the possible answers, the mentioned rule / property in the set-builder from must result into exactly the same elements given in the roster form.

(a) $\{x : x \text{ is an integer and } -4 \le x \le 5\}$

(b) $\{x : x = n^2 - 1, \text{ where n is a natural number and } 1 \le n \le 6\}$

(c) $\left\{x : x = \dfrac{n}{n+1},\ \text{where n is a natural number and } 1 \le n \le 6\right\}$

(d) $\{x :\ x = n^3 + n,\ n \in Z - Z^-,\ n \le 5\}$

(e) $\{x : x \text{ is a root of } x^2 - 4 = 0\}$

(f) $\{x : x = 7m, \text{ where } m \in N, 1 < m \le 12\}$ or $\{x : x = 7n, n \in N \text{ and } 7 < x < 90\}$

(g) $\{x : x \text{ is a prime number and } 50 < x < 100\}$

(h) $\{x : x \in N, x \text{ is equal to 1 or multiple of } 5\}$

(i) $\left\{x : x = \dfrac{1}{n^2}, n \in N\right\}$ (j) $\{x : x^2 \le 4, x \in Z\}$

(k) $\{x : x^2 \le 9, x \in N\}$ (l) $\{x : x = 3^n, n \in N, n \le 5\}$

Q04. $i - c$, $ii - a$, $iii - b$, $iv - d$.

Q05. If an element $x$ is present in set A then we write $x \in A$; if the element x is not present in A then we write $x \notin A$.

(a) $\in$ (b) $\notin$ (c) $\in$ (d) $\notin$

Q06. (a) The natural nos. less than 6 except 4 means the natural nos. that we will take are : 1, 2, 3, 5. Now we have to consider the cubes of 1, 2, 3 and 5 to write the set Y.
Hence, $Y = \{1, 8, 27, 125\}$.

(b) As x is an integer and $x^2 \le 9$ so, x can take the values $x = 0, \pm 1, \pm 2, \pm 3$.
Hence, the required set is $\{-3, -2, -1, 0, 1, 2, 3\}$.

(c) The vowels which precede q are $a, e, i, o$. Hence the required set is $\{a, e, i, o\}$.

(d) As $n \in Z^+$ and $n < 4$ so, $n = 1, 2, 3$.
Then the value of p will be $\dfrac{1}{2\times 1} = \dfrac{1}{2}, \dfrac{1}{2\times 2} = \dfrac{1}{4}, \dfrac{1}{2\times 3} = \dfrac{1}{6}$.
Hence, the set will be given by $\left\{\dfrac{1}{2}, \dfrac{1}{4}, \dfrac{1}{6}\right\}$.

Q07. (a) There is no even integer whose cube is odd. Therefore, we will have a null set i.e., $\{\ \}$ or $\phi$.

(b) Recall that, a rational number can be written as the quotient of two integers. But an irrational number can not be written as the quotient of two integers.
Therefore, the required set is $\{x : x \in T\}$ or $\{x : x \text{ is real and irrational number}\}$.

Q08. Given that $X = \{1, 2, 3, 4, 5, 6, 7\}$, let $A = \{n \in X \text{ but } 2n \notin X\}$
Note that, if we put $n = 1, 2 \text{ or } 3 \in X$, then $2n = 2, 4 \text{ or } 6 \in X$.
Whereas if we put $n = 4, 5, 6 \text{ or } 7 \in X$, then $2n = 8, 10, 12 \text{ or } 14 \notin X$.
Clearly, $A = \{4, 5, 6, 7\}$.

Q09. The elements of the required set are not even integers (as cube of an even integer is always an even integer).
That is, the elements of the required set must be all positive odd integers.
Hence the set builder form of the required set is $\{2m + 1 : m \ge 0, m \in Z\}$.

## Exercise 1.2

Q01. (a) Since $\{x : x \text{ is an integeral root of } x^2 - 2x + 1 = 0\} = \{1\}$ has only one element. Therefore, it is a singleton set.

(b) Let $A = \{0, \{\}\}$. The cardinal number of set A is 2. So it is not singleton set.

Q02. We have $A = \{0\}$, $B = \{x : x > 15 \text{ and } x < 5\} = \{\ \} = \phi$, $C = \{x : x - 5 = 0\} = \{5\}$ and $D = \{x : x^2 = 25\} = \{-5, 5\}$.

So it is clear that no pair of the sets are equal (as for equal sets A and B *say*, every element of set A must be present in B and every element of set B must be in A).

Q03. (a) Given $A = \{x : x \in Z \text{ and } x^2 \le 4\}$, $B = \{x : x \in R \text{ and } x^2 - 3x + 2 = 0\}$

So, $A = \{-2, -1, 0, 1, 2\}$, $B = \{1, 2\}$.

$\therefore A \neq B$ $[\because n(A) \neq n(B)$]

(b) Given A = {x : x is a letter in the word FOLLOW}, B = {y : y is a letter in the word WOLF}.

So, A = {F, O, L, W} and B = {W, O, L, F}

$\because n(A) = n(B)$, for all $x \in A$, $x \in B$ and for all $y \in B$, $y \in A$

$\therefore A = B$.

Q04. (a) $\{x : x \in R, -5 < x \le 9\}$

(b) $[-3, 7]$

(c) $\{x : x \in R,\ a < x < a\}$

(d) $\because n(A) = 3$ $\therefore$ No. of subsets of $A = 2^3 = 8$.

(e) $\phi, \{5\}, \{6\}, \{5, 6\}$

Q05. (a) $\{x : x \in R,\ -3 < x < 0\}$ (b) $\{x : x \in R,\ 6 \le x \le 12\}$

(c) $\left\{x : x \in R,\ -\frac{5}{2} \le x < 5\right\}$ (d) $\{x : x \in R,\ 1 < x \le 3\}$.

Q06. (a) Let $A = \{a, b\}$.

Since n(A) = 2 so, total number of subsets of A = $2^2 = 4$.

$\therefore$ subsets of $A = \phi, \{a\}, \{b\}, \{a, b\}$.

(b) Let A = {1, 2, 3}.

Since n(A) = 3 so, total no, of subsets of A = $2^3 = 8$.

$\therefore$ subsets of $A = \phi, \{1\}, \{2\}, \{3\}\ \{1, 2\}, \{2, 3\}, \{3, 1\}, \{1, 2, 3\}$.

(c) Let $A = \phi$.

Since n(A) = 0 so, total number of subsets of A = $2^0 = 1$.

$\therefore$ subsets of $A = \phi$

(d) Let A = {–1, 0, 1}.

Since n(A) = 3 so, total no. of subsets of A = $2^3 = 8$.

$\therefore$ subsets of $A = \phi, \{-1\}, \{0\}, \{1\}\ \{-1, 0\}, \{0, 1\}, \{1, -1\}, \{-1, 0, 1\}$.

Q07. We have X = {M, O, R}

$\therefore$ Subsets of X are $\phi$, {M}, {O}, {R}, {M, O}, {O, R}, {R, M}, {M, O, R}.

Q08. Here A = {I, S, W} *say*.

Clearly total number of elements in set A is 3. So, no. of proper subsets $= 2^3 - 1 = 7$.

Also the subsets are $\phi$, {I}, {S}, {W}, {I, S}, {S, W}, {W, I}, {I, S, W}.

Q09. (a) $\in$ (b) $\notin$ (c) $\subset$ (d) $=$ (e) $=$ (f) $\not\subset$ (g) $\subset$ (h) $\subset$ (i) $\notin$ (j) $\notin$ (k) $\notin$

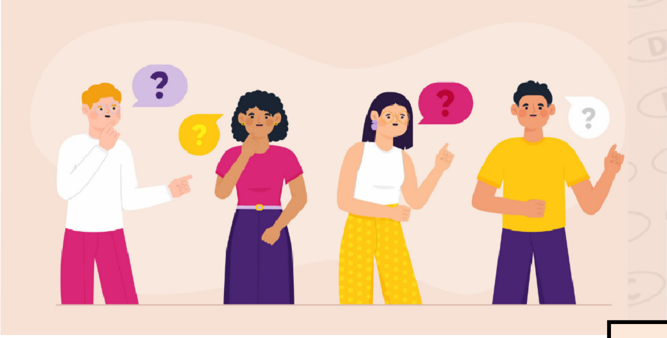
*Can you think, if more different symbols can be used in some of the cases?*

Q10. (a) It is an infinite set because circle is a collection of infinite points whose distances from the centre is constant.

(b) It is a finite set as $\{m : m \in N \text{ and } m \text{ is an even prime number}\} = \{2\}$.

(c) Here the set will be {101, 102, 103, …}, which is an infinite set.

(d) There are infinite lines which will be parallel to x-axis. Hence the given set is infinite set.



# MULTIPLE CHOICE TYPE QUESTIONS

For 2027 Exams - Mathematics (041) - Class 11

**SOLUTIONS**

## ☑ Chapter 01
## Sets Theory

01. A two digit number begins from 10 and ends at 99.
Clearly, a number so that the sum of its digits is one is only 10.
Therefore the required set will be $\{10\}$.

02. Since $n(A)=1$, so A is singleton set.

03. $X-Y=\{2,4\}$.

04. Since $A\cup U=U$ so, clearly $A\cup U=A$ is **not** true.

05. Only $A\cap A'=\phi$ is true.

06. As $\{1,2,3\}\cap\{3,4,5\}=\{3\}\neq\phi$ so, this pair of sets is non disjoint.

07. For $x^2+x-2=0$, we get $(x+2)(x-1)=0 \quad \Rightarrow x=1\in N$
Then set $A=\{1\}$. Clearly, $n(A)=1$.

08. Since $A=\{1\}$. Clearly, $n(A)=1$.
Hence number of subsets of $A=2^1=2$.

09. $A\cup B=\{1,2,3,4,5\}\cup\{4,5,6,7\}=\{1,2,3,4,5,6,7\}$.

10. $A\cap B=\{1,2\}\cap\{3,4,5,6,7\}=\phi$.

11. $A-B=\{1,2,3\}$.

12. $n(A\cup B)=n(A)+n(B)-n(A\cap B)$
$\Rightarrow n(A\cup B)=3+6-2=7$.

13. $A'=U-A=\{2,4,6,9\}$.

14. For $A=\{\phi\}$, $n(A)=1$. So, A is singleton set.

15. Here $n=1,2,3 \quad \Rightarrow n+6=7,8,9$.
Therefore, the required set is $\{7,8,9\}$.

16. The set builder from will be $\{x : x \text{ is a prime natural no. between 6 and 20}\}$.

17. Set builder form of the null set can be represented by $\{x : x\neq x\}$.

18. $A\cap(A\cap B)'=A\cap(A'\cup B')=(A\cap A')\cup(A\cap B')$
$=\phi\cup(A\cap B')=(A\cap B')=A-B$.

19. Since $A\subset B$ implies, all the elements of A are in B i.e., set A is contained in B.
Moreover $B'$ means all elements of universal set which are not in B.
Therefore, $A\cap B'=\phi$.

20. Since $n(A\cup B)=n(A-B)+n(A\cap B)+n(B-A)$
$\Rightarrow 18=5+n(A\cap B)+3$
$\therefore\ n(A\cap B)=10$.

21. $A\cap(A\cup B)=A$.

22. Since $n(A\cup B)=n(A)+n(B)-n(A\cap B)$
Clearly, $n(A\cup B)$ is maximum when $n(A\cap B)$ is minimum i.e., $n(A\cap B)=0$.
Hence, $n(A\cup B)=5+7-0=12$.

23. Recall that a null set, $\phi$ has no elements. So, $n(\phi)=0$.

24. Here $n(A)=5$.
Then the number of proper subsets of $A = 2^n - 1 = 2^5 - 1 = 31$.
25. $(A-B)\cap(B-A)=\phi$.
26. $X=\{0, 49, 490, 4067, 32732, ...\}$ and $Y=\{0, 49, 98, 147, 196, ..., 490, ...\}$
Note that all the elements of X are in Y. Therefore, $X\subset Y$.
27. Since $A\cap B=B$. that means, all the elements of B are in A. Therefore, $B\subset A$.
28. $X\cap(X\cup Y)'=X\cap(X'\cap Y')=(X\cap X')\cap(X\cap Y')=\phi\cap(X-Y)=\phi$.
29. $A'=U-A$.
30. For the disjoint pair of sets A and B, we always have $A\cap B=\phi$.
Therefore, the statement given in option (d) is **not** correct.
31. $A'\cup\big((A\cup B)\cap B'\big)=A'\cup\big((A\cap B')\cup(B\cap B')\big)=A'\cup\big((A\cap B')\cup\phi\big)$
$=A'\cup(A\cap B')=(A'\cup A)\cap(A'\cup B')=U\cap(A'\cup B')$
$=U\cap(A\cap B)'$ [As A and B are disjoint sets so, $A\cap B=\phi$
$=U\cap(\phi)'=U\cap U=U$.

32. {C, L, O}.
33. not a well defined collection.
34. $\{\phi\}$.
35. an infinite set.
36. $\phi$.
37. $[-4,-2]$.

## ☑ Chapter 02
## Relations & Functions

01. For $f(x)$ to be defined, we must have $x^2-5x-6>0$ i.e., $(x-6)(x+1)>0$
That is, $x\in(-\infty,-1)\cup(6,\infty)$ or $x\in\mathbb{R}-[-1, 6]$, which is the domain of given function.
02. $|x|\geq 3$ implies, $x\leq -3$ or $x\geq 3$ i.e., $x\in(-\infty,-3]\cup[3,\infty)$.
03. Note that $n(A)=4$, $n(B)=3$.
Then no. of functions defined from A to B is $3^4=81$.
04. Since a greatest integer function always results in an integral value, for all real inputs.
Therefore, the required range is $Z$.
05. Since R is defined from A to B, so, $R\subseteq A\times B$.
Therefore a relation having maximum no. of elements is $R=A\times B$.
06. Here $n(A)=2$, $n(B)=3$.
Then the total number of relations from A to B will be $2^{2\times 3}=64$.
07. $A=\{1,2,3\}$, $B=\{4,5,6\}$; $R':B\to A$ defined as $R'=\{(x,y): x\in B,\ y\in A;\ x \text{ is divisible by } y\}$.
Then $x=4,5,6;\ y=1,2,3$.
Therefore, the roster form of $R'$ is given by $\{(4,1),(4,2),(5,1),(6,1),(6,2),(6,3)\}$.
08. The given graph represents Signum function.
09. Here $n(A)=2$, $n(B)=3$.
Then the total number of functions from B to A will be $2^3=8$.
10. The given function is defined only when $[x]\neq 0$. That means, $x\notin[0, 1)$.
Hence, the domain is $\mathbb{R}-[0, 1)$.
11. The function f is defined when $|x|-x\geq 0$ i.e., $|x|\geq x$. This will be true for all real values of x.
Hence, the function f is defined when $x\in\mathbb{R}$.
12. Since $|x|\geq 0\ \ \forall\ x\in\mathbb{R}$
That is, $0\leq|x|<\infty\ \ \forall\ x\in\mathbb{R}$
$\Rightarrow 0\geq -|x|>-\infty$

Then, $-\infty < f(x) \le 0$.
Therefore, the range of $f(x)$ is given by $(-\infty, 0]$.

13. Since $f(x)$ is defined for $-5 \le x \le 5$.
So, $0 \le x^2 \le 25 \quad \Rightarrow 0 \ge -x^2 \ge -25$
$\Rightarrow 25 \ge 25 - x^2 \ge 0 \quad \Rightarrow 0 \le \sqrt{25 - x^2} \le 5$
$\Rightarrow 0 \le f(x) \le 5$.
Therefore, the range of $f(x)$ is [0, 5].

14. The function is $f(x)$ defined when $1 + |x| \ne 0$, which is true for all $x \in \mathbb{R}$.
Hence, $\text{dom.}(f) = x \in \mathbb{R}$.

15. Since $S \subset A \times B$ so, S is a relation from A to B.
Similarly, S is a function from A to B. Since each element of A has a unique image in B.
Therefore, S is a relation and function both.

16. For the domain of R, we need to write those values of x which satisfy $x \in Z$ and $|x| \le 3$.
Clearly, $x = \pm 3, \pm 2, \pm 1, 0$.
Hence, the domain of R is $\{\pm 3, \pm 2, \pm 1, 0\}$.

17. Here $n(A) = 3,\ n(B) = 2$.
Then no. of non-empty relations defined from A to B is $2^{3\times 2} - 1 = 63$.

18. Since number of elements in an empty set is 0.
Therefore, the number of relations defined on an empty set is $2^{0\times 0} = 1$.

19. The domain of function $f(x)$ is $x \in R$ (real nos.).

20. Since $|x| \ge 0 \ \forall\ x \in \mathbb{R}$
That is, $0 \le |x| < \infty \ \forall\ x \in \mathbb{R}$
$\Rightarrow 0 \ge -|x| > -\infty \quad \Rightarrow 3 \ge 3 - |x| > -\infty$
Then, $-\infty < f(x) \le 3$.
Therefore, the range of $f(x)$ is given by $(-\infty, 3]$.

21. For the function to be defined, $-x > 0$ i.e., $x < 0$.
Therefore, the domain is $x \in \mathbb{R} - [0, \infty)$.

22. Note that $(B \cap C) = \{5, 6, 7\}$. That is, $n(A) = 2,\ n(B \cap C) = 3$.
Therefore, $n[A \times (B \cap C)] = 2 \times 3 = 6$.

23. Since $|x - 2| \ge 0 \ \forall\ x \in \mathbb{R}$
That is, $0 \le |x - 2| < \infty \ \forall\ x \in \mathbb{R}$
$\Rightarrow 0 \ge -|x - 2| > -\infty$
Then, $-\infty < f(x) \le 0$.
Therefore, the range of $f(x)$ is given by $(-\infty, 0]$.

24. $(A - B) \times (B - C) = \{1\} \times \{4\} = \{(1, 4)\}$.

25. $x \in A = \{1, 2, 3, 4, 5, 6, 7, 8\}$, also $y \in A$; $R : A \to A$
$\therefore R = \{(1,3), (2,6)\}$

26. $A = \{1, 2, 3\}$, $B = \{4, 6, 9\}$; $R : A \to B$ and $R = \{(x, y) : x \text{ is greater than } y\}$.
Clearly, $x \in A,\ y \in B$. That implies, $R = \phi$.

27. If R be a relation from a set A to a set B, then $R \subseteq A \times B$.

28. Given that $2f(x) - 3f\left(\frac{1}{x}\right) = x^2,\ (x \ne 0)$,

# ASSERTION REASON Type Questions

**SOLUTIONS**

## ☑ Unit-I

01. Both the sets given in Assertion (A) are having exactly same elements.
Therefore, they are equal sets. So, A and R both are true and R is the correct explanation of A.

02. Both A and R are true and R is not the correct explanation of A.

03. Since $2^n \neq 5$ for any value of $n \in Z - Z^-$. Therefore, A is false and R is true.

04. Note that, the set will be $\{\text{February}\}$, which is a singleton set.

05. Note that two integers 'a' and 'b' ate coprime if their only positive integer divisor is 1.
Further, coprime numbers are also known as 'relatively prime' or 'mutually prime' numbers.
Clearly, Assertion is true. Also Reason is true however it is not correct explanation of Assertion.

06. Note that, Reason is not true as $\{a, b\}$ is a set. Also, Assertion is true.

07. Domain of $f(x) = \sqrt{x-4}$ is $x \geq 4$. That means, the statement A is false. Further R is true.

08. Domain of $f(x) = \dfrac{1}{9-x^2}$ is $x \in R - \{3, -3\}$. That means, the statement A is false.
Further note that, R is true.

09. Note that $(1, b), (1, c) \in f$. Since '1' is related to 'b' and 'c' both. So, f is not a function.
Hence, both A and R are true and R is the correct explanation of A.

10. Note that, $m + 2 = 5 \Rightarrow m = 3,\ m - n = 0 \Rightarrow m = n$ i.e., $m = 3 = n$.
Hence, Assertion and Reason both are true and Reason is correct explanation of Assertion.

11. Note that, $f(p) = 2p + 5 = 9$ implies, $p = 2$. So, Assertion is false. Further the Reason is true.

12. $\cos x = -\dfrac{1}{3}$

$\Rightarrow 2\cos^2\dfrac{x}{2} - 1 = -\dfrac{1}{3}$

$\Rightarrow \cos^2\dfrac{x}{2} = \dfrac{1}{3}$

$\Rightarrow \cos\dfrac{x}{2} = -\dfrac{1}{\sqrt{3}}$ $\quad \left[\because \pi < x < \dfrac{3\pi}{2} \quad \therefore \dfrac{\pi}{2} < \dfrac{x}{2} < \dfrac{3\pi}{4}\right.$

Hence, both A and R are true and R is the correct explanation of A.

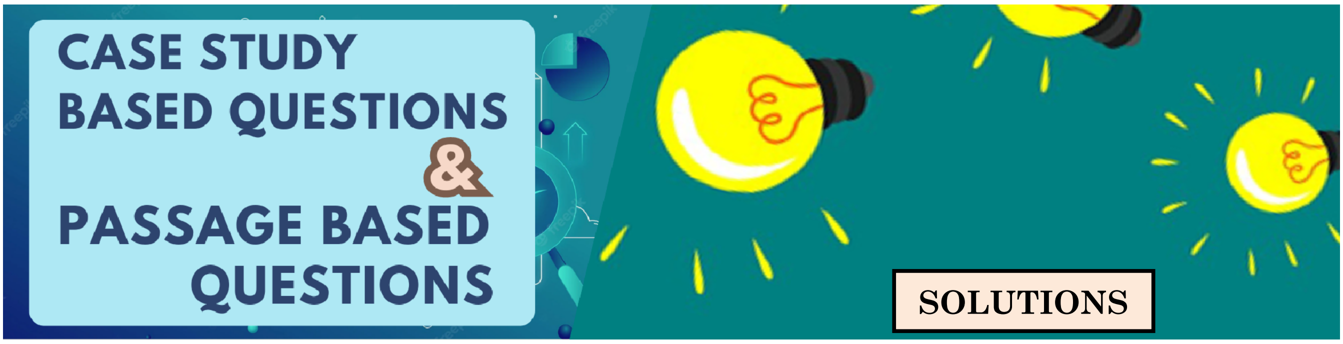
13. $\sin(-390^\circ) = \sin(-360^\circ - 30^\circ) = -\sin 30^\circ = -\dfrac{1}{2}$. So, A is true.

Also, note that $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$. So, R is false.

14. $\dfrac{11}{7}$ radians $= \left(\dfrac{11}{7} \times \dfrac{180}{\pi}\right)^\circ = \left(\dfrac{11}{7} \times \dfrac{180}{22} \times 7\right)^\circ = 90^\circ$. So, A is true.
Further note that R is also correct and it explains the statement in A.

15. As $\cos^2 x = 1 - \sin^2 x$

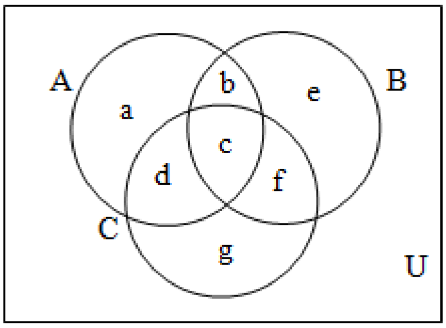
$\Rightarrow \cos^2 x = 1 - \left(-\dfrac{1}{3}\right)^2 = \dfrac{8}{9} \qquad \therefore \cos x = \pm\dfrac{2\sqrt{2}}{3}$




☑ **Chapter 01**

**Sets Theory**

Q01. (i) Let A, B and C denote the set of students who took English, Hindi and Sanskrit respectively.
Consider the Venn diagram.




We have $a+b+c+d=18$, ...(a)

$b+c+e+f=23$, ...(b)

$c+d+f+g=24$, ...(c)

$c+f=13$, ...(d)

$b+c=12$, ...(e)

$c+d=11$, ...(f)

$c=6$ ...(g)

On solving these equations simultaneously, we get :

$a=1, b=6, c=6, d=5, e=4, f=7, g=6$

Therefore, total number of students $= a+b+c+d+e+f+g=35$.

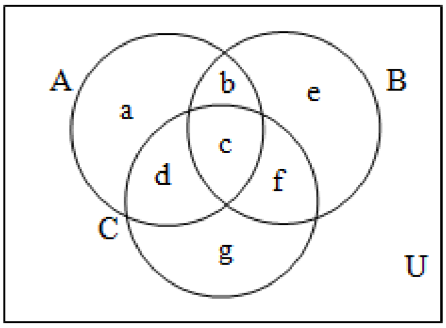
(ii) Number of students who took Sanskrit but not Hindi $= d+g=11$.

(iii) Number of students who took exactly one of the three subjects $= a+e+g=11$.

(iv) Number of students who took exactly one of the three subjects $= b+d+f=18$.

(v) Number of students who took Hindi but not Sanskrit $= b+e=10$.

Q02. (i) Let A, B and C be the set of families who buy milk of brand A, B and C respectively.
We have been given $n(A)=40\%$, $n(B)=20\%$, $n(C)=10\%$, $n(A\cap B)=5\%$, $n(B\cap C)=3\%$, $n(C\cap A)=4\%$, $n(A\cap B\cap C)=2\%$.
Consider the Venn diagram shown.




We have $a+b+c+d=40\%$,

$b+c+e+f=20\%$,

$c+d+f+g=10\%$,

$b+c=5\%$,

$c+f=3\%$,

$c+d=4\%$,

$c=2\%$.

On solving all these equations simultaneously, we get :

$a=33\%,\ b=3\%,\ c=2\%,\ d=2\%,\ e=14\%,\ f=1\%,\ g=5\%$.

Therefore, the number of families buying brand A only $= a = 33\%$ of 10000.

$= 10000\times 33\% = 3300$.

(ii) Number of families buying exactly two brands of milk $= b+d+f = 6\%$ of 10000

$= 10000\times 6\% = 600$.

(iii) Number of families buying exactly one milk brand $= a+e+g = 52\%$ of 10000

$= 10000\times 52\% = 5200$.

(iv) Number of families buying brands A and C but not B $= d = 2\%$ of 10000

$= 10000 \times 2\% = 200$.

(v) Number of families buying at least one of the brands A, B and C

$= n(A \cup B \cup C) = (a+b+c+d+e+f+g) = 60\%$ of 10000

$= 10000 \times 60\% = 6000$.

Q03. Let Y and B denote the set of students using YouTube videos and Books respectively.

Given that $n(Y) = 150$, $n(B) = 225$, $n(Y \cap B) = 100$.

(i) Since $n(Y \cup B) = n(Y) + n(B) - n(Y \cap B)$

$\Rightarrow n(Y \cup B) = 150 + 225 - 100 = 275$.

There are 275 students who are using either YouTube videos or books.

(ii) No. of students who are neither using YouTube videos nor books

$= n(U) - n(Y \cup B) = 600 - 275 = 325$.

(iii) No. of students who are using YouTube videos only $= n(Y) - n(Y \cap B) = 150 - 100 = 50$.

(iv) No. of students who are using Books only $= n(B) - n(Y \cap B) = 225 - 100 = 125$.

(v) Maximum no. of students who will use YouTube video or books can be obtained when none of the students are using both YouTube videos and books.

That is, when $n(Y \cap B) = 0$.

Hence, $n(Y \cup B) = 150 + 225 - 0 = 375$.

Q04. Let $n(A) = m$ and $n(B) = n$.

Given $n(A) + n(B) = 9$ i.e., $m + n = 9$

Also, total number of subsets of $A = 2^m$ and total number of subsets of $B = 2^n$.

Given that $2^m : 2^n = 8 : 1$ i.e., $\frac{2^m}{2^n} = \frac{2^3}{1}$ i.e., $2^{m-n} = 2^3$

$\Rightarrow m - n = 3$

(i) On solving $m + n = 9$ and $m - n = 3$, we get $2m = 12$ $\therefore m = 6$.

(ii) On solving $m + n = 9$ and $m - n = 3$, we get $2n = 6$ $\therefore n = 3$.

(iii) Since $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

The value of $n(A \cup B)$ will be maximum, when $n(A \cap B)$ will be minimum.

Also the minimum value of $n(A \cap B) = 0$.

So, maximum value of $n(A \cup B) = n(A) + n(B) = 6 + 3 = 9$.

(iv) The value of $n(A \cup B)$ will be minimum when $n(A \cap B)$ will be maximum.

Also the maximum value of $n(A \cap B) = 3$.

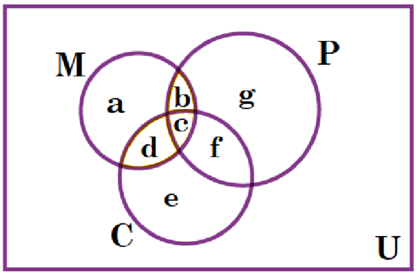
So, minimum value of $n(A \cup B) = n(A) + n(B) - 3 = 6 + 3 - 3 = 6$.

(v) Given $B \subset A$ so, $A \cap B = B$ i.e., $n(A \cap B) = n(B)$.

Clearly, $n(A \cap B) = 3$.

Q05. Let M, P and C denote the set of students who had taken Mathematics, Physics and Chemistry respectively.

Consider the venn diagram shown aside.




Given that $n(M) = a + b + c + d = 21$,

$n(P) = b + c + g + f = 16$,

$n(C) = c + d + e + f = 15$,

$n(M \cap C) = c + d = 7$,

$n(M \cap P) = b + c = 12$,

$n(P \cap C) = c + f = 5$,

$n(M \cap P \cap C) = c = 4$.

On solving the above equations, we get $c = 4$, $f = 1$, $b = 8$, $d = 3$, $e = 7$, $g = 3$, $a = 6$.

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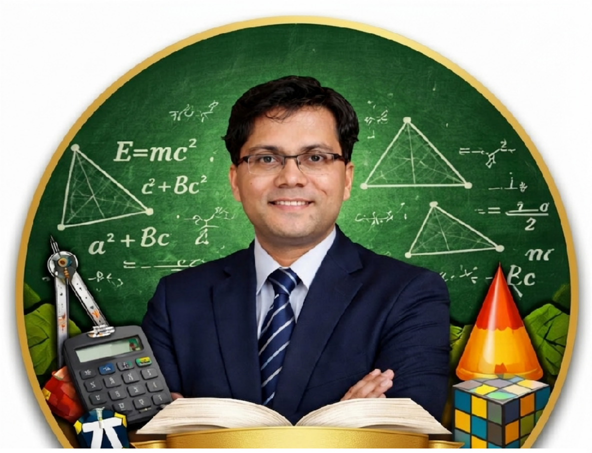



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## ABOUT THE AUTHOR

O.P. GUPTA having taught math passionately over a decade, has devoted himself to this subject. Every book, study material or practice sheets, tests he has written, tries to teach serious math in a way that allows the students to learn math without being afraid. Undoubtedly his mathematics books are best sellers on amazon and Flipkart.

His resources have helped students and teachers for a long time across the country. He has contributed in CBSE Question Bank (issued in April 2021). Mr Gupta has been invited by many educational institutions for hosting sessions for the students of senior classes. Being qualified as an electronics & communications engineer, he has pursued his graduation later on with mathematics from University of Delhi due to his passion towards mathematics. He has been honored with the prestigious INDIRA AWARD by the Govt. of Delhi for excellence in education.




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